Two "Log Spiral" Devices

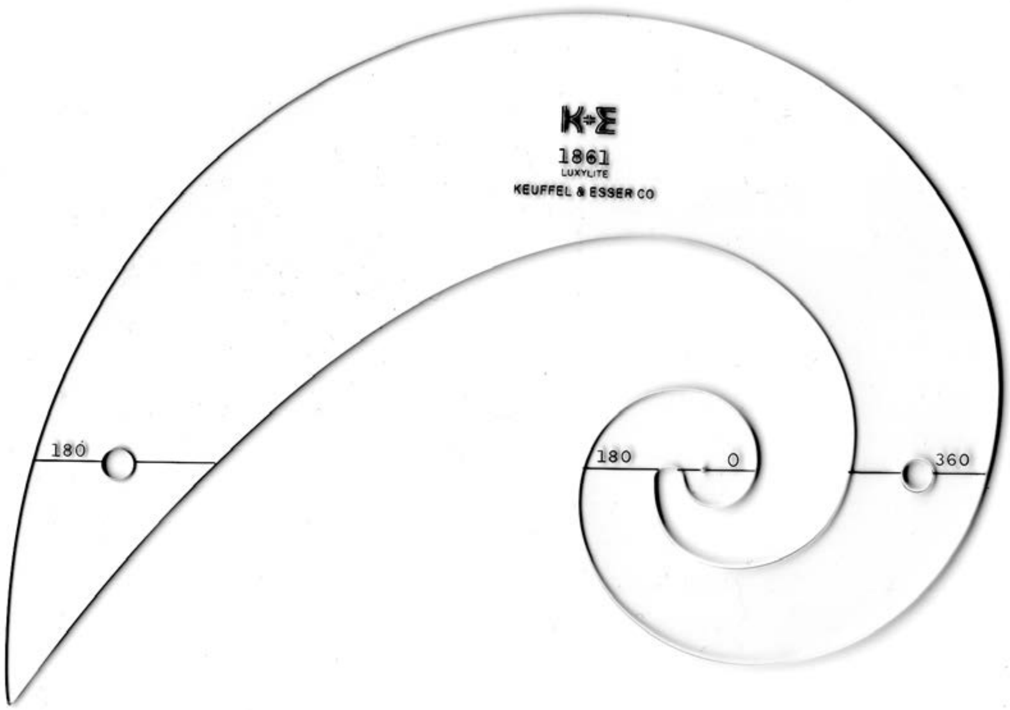

Figure 1. The K&E Log Spiral Device.

Rodger Shepherd

Introduction

The K&E Logarithmic Spiral Curve and the Tyler Slide Rule are two analog calculating devices that appear unrelated but are actually based on the same obscure principle. This article describes the two devices and explores the underlying concept common to both devices.

The K&E Logarithmic Spiral Curve

This device appears to have been championed by William Cox, a pioneer in the early development of K&E slide rules.

The Log Spiral is a flat, complex spiral template resembling a draftsman's French curve. (See Figure 1.) The maximum dimension is 20.13 cm. The Log Spiral is listed in K&E catalogs from 1895 through 1955. Judging by the catalog number (57 1000) on my Log Spiral manual, the device may have been offered into the 1960s. At various times the Log spiral was available in celluloid, hard rubber, xylonite, and luxylite.

The K&E catalogs describe the Log Spiral as follows: "This curve is constructed on mathematical principles and contains every curve within the limit of its size. It is a tool of large scope and useful also for various calculations."

In the standard K&E catalogs, the Log Spiral appeared among the French Curves, and the device was not listed in the K&E special catalogs on calculating devices. Perhaps for these reasons the device does not seem to have gained much recognition among slide rule collectors.

The Log Spiral manual was authored by William Cox. The final page of the manual is devoted to four drafting applications; however, the first seven of the manual's eight pages are devoted to the mathematical basis of the log spiral and how to use it to multiply and divide. Thus Cox appeared to be most intrigued with the mathematical origins and applications of the Log Spiral. These will be discussed below.

Some Features of the Log Spiral

In the manual for log spiral scale, Cox provides a formal explanation of the derivation of the log spiral scale and how it can be used to solve proportions and carry out multiplication and division. I will attempt to provide a less formal but more easily understood explanation.

By definition a circle has a radius of constant length. In contrast to a circle, a spiral has radii (technically known as radii vectores) which increase as the spiral expands. In the log spiral, the radii expand exponentially. This point is illustrated in Figure 2, which compares a segment of Cox's log spiral with a segment of circle—both of which share the same center. If the radius of the circle is arbitrarily assigned a length of 1.0 and the arc of the circle is divided into four equal parts, it can be seen that the length of the radii vectores progress geometrically with the powers of 1.24. (For clarity, I have arbitrarily limited the significant digits to three.) This relationship applies regardless of where one starts (i.e., which radius vector is designated line PA).

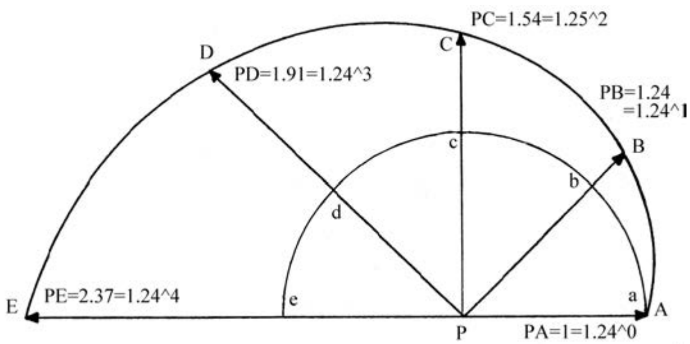

Figure 2.

Cox points out that this geometric progression can be used to solve proportions. For example, when a log spiral has the same pitch as Cox's log spiral, then $BP/AP = CP/BP = DP/CP = EP/DP = 1.24$ for all pairs of radii vectorii that are separated by 45°. Similarly $CP/AP = DP/BP = EP/CP = 1.54$ for all pairs of radii vectores separated by 90°. Thus, if $X/Y = Z$, and radii vectores with length X and Y are located, then the angle between the two radii can be transferred anywhere else on the same log spiral to identify pairs of radii that satisfy the same proportion.

Cox also explains how the log spiral can be used to carry out multiplication and division. For example, to multiply 1.24×1.91, one would find point B (See Figure 2) on the log spiral where the radius vector is 1.24 (or 12.4 or 124) units long. (The absolute size of the units does not make any difference.) Using the same scale, one would find the point A where the radius vector is 1.00 (or 10.0 or 100, respectively) units from the center P. Again using the same scale, one would find the Point D where the radius vector was 1.91 units long. The three radii vectores AP, BP, and DP would be drawn. The arc ab on the circle would be transferred with dividers to point d in order to find point e. Finally, the radius vector EP would be measured using the original scale. The length of the radius vector EP, i.e., 2.37 original units, is the product (rounded to three significant digits) of 1.24×1.91. In order to divide, one reverses the process.

An interesting and relevant feature of the log spiral is that the radius vector, which lies halfway between two other radii vectores, is their geometric mean, i.e., the square root of their product. For example (again referring to Figure 2), BP is the square root of $AP \times CP$, CP is the square root of $BP \times DP$, and so on. If one member of the vector pair is assigned the value 1.0, the second member becomes the square of the geometric mean. As will be seen, the Tyler slide rule takes advantage of this relationship.

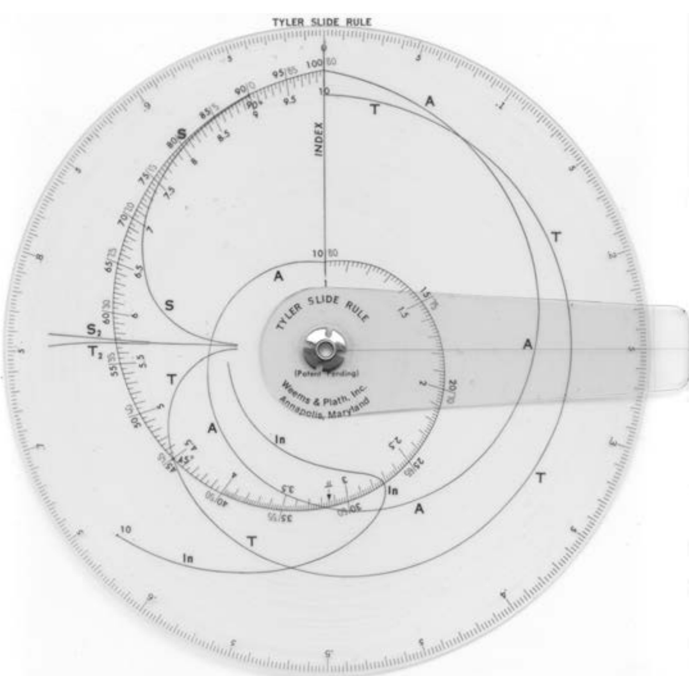

Figure 3.

The Tyler Slide Rule

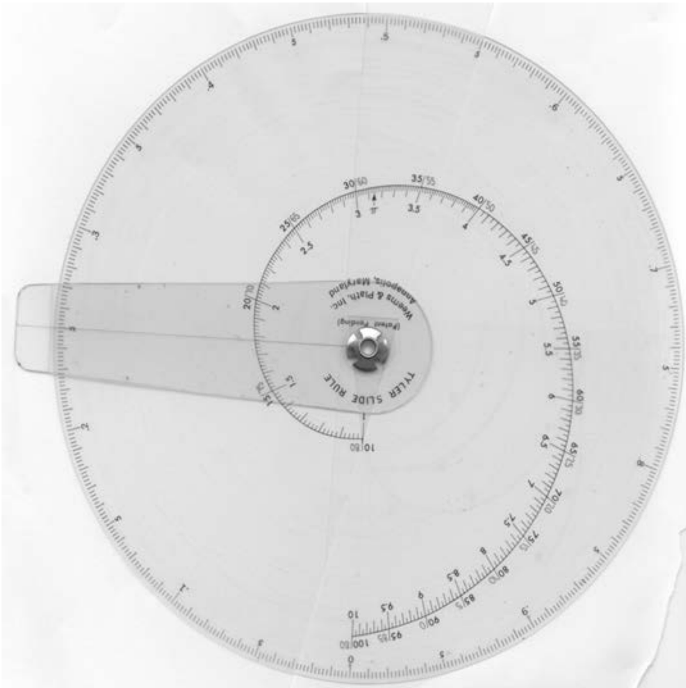

Figure 4.

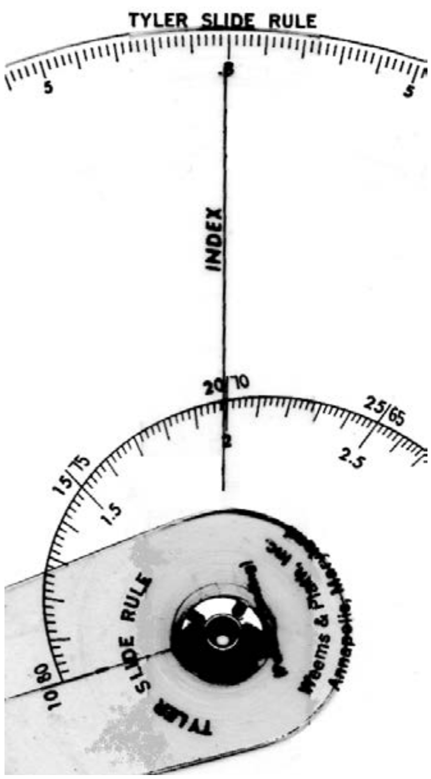

Figure 5.

John Tyler was a professor of mathematics at the United States Naval Academy.

The Tyler slide rule consists of a 9.5-cm clear (celluloid?) disc and clear cursor arm, both mounted on a white plastic base plate that is 21.5 cm square. (See Figure 3.) Complete instructions are printed on the back of the base plate.

My specimen indicates that it was made by "Weems & Plath of Annapolis, Maryland". A later specimen (from the collection of Bruce Reichelt) is marked "Weems System of Navigation, a Division of Jeppesen & Co, Annapolis, Maryland". This information helps date the slide rule, because in the mid-1950s Weems was still using an office at Weems & Plath . (Personal communication, Alan Morris.) Thus the Tyler slide rule was in production sometime after the mid-50s.

As can be seen in Figure 3, all the scales (except for one) are spiral and present a daunting array. However, the structure becomes clearer when the scales on the base plate are masked. (Figure 4) It is now apparent that three of the six scales are mounted on the disk. One of these scales is a one-cycle linear scale marked along the edge of the disk. The other two are a one-cycle logarithmic scale and a bi-directional scale marked in degrees. These two scales are marked on the spiral. The index line on the cursor can be used to read (on the outer scale) the logarithm of any number on the log spiral scale. Also, the index line on the cursor and the index line on the base plate essentially serve as a caliper that can be used to transfer scale segments. For example, to multiply 2×3 the disk is rotated so that 2 on the spiral scale is aligned with the index on the base plate. (See Figure 5.) Next, the cursor is set at 0 on the spiral scale. If the cursor arm is held fixed, and the disc is rotated so that 3.0 overlies the index line on the cursor, then the product (6.0) overlies the index line on the base plate. (See Figure 6.) This is essentially the same principle used in the Gilson circular slide rules; an important comparison, because it serves to point out that the log scale does not have to be spiral in order to function in this way.

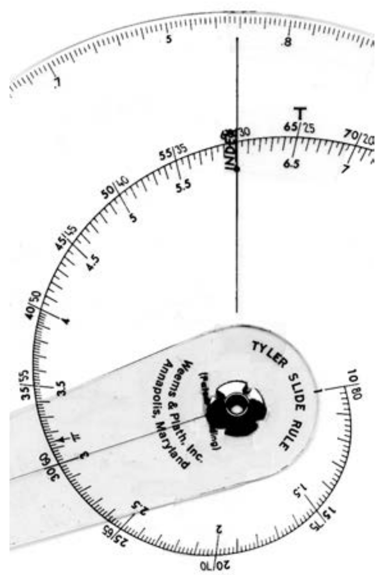

Figure 6.

One possible advantage of displaying the logarithmic scale along a spiral is that this serves to expand the intervals and increase the precision at the high ("right") end of the scale. For example, on the Tyler the interval $9.0 - 10.0$ is approximately the same absolute length as the interval $1.0 - 1.4$. On a conventional linear slide rule, $9.0 - 10.0$ is the same absolute length as $1.0 - 1.1$. However, this is an arguable advantage, since the entire scale would be expanded if it were located at the edge of the disc.

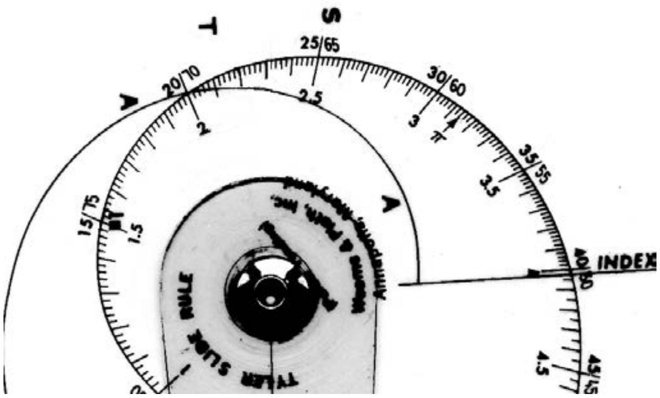

Figure 7.

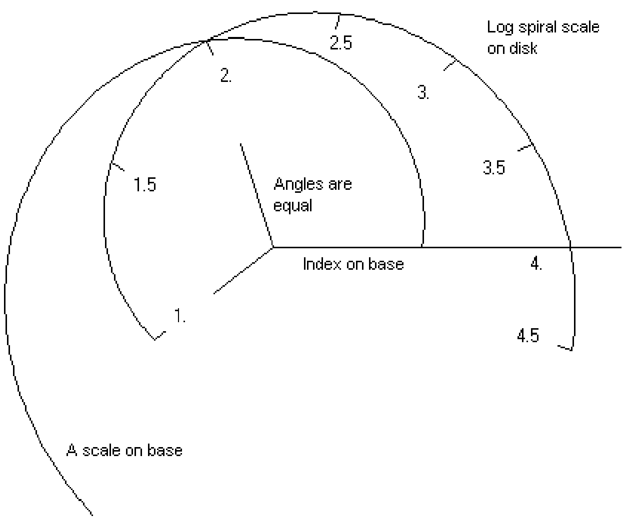

Figure 8.

Much more intriguing is the *A* scale, which is used to determine squares and square roots. The *A* scale is on the base plate and is the mirror image of the log spiral scale on the disk. In Figure 7, the disk is set so that the point 2.0 on the log spiral scale is directly over the corresponding point on the *A* scale. It can be seen that the square of 2.0 (i.e. 4.0) can be read off the log spiral scale where it crosses the index line on the base plate. This operation is definitely dependent on the spiral shape of the two scales. Figure 8 shows how the two matching but reversed spirals serve to locate the geometric mean between 1.0 and 4.0 or, vice versa, identify the square of 2.0. This Figure also illustrates how using the Tyler often depends on identifying the intersection of two lines that are nearly parallel.

There are four sets of trigonometric curves: S and T for under $10°$, plus S and T for over $10°$. The trigonometric scales are designed so that the function of the angle is read on the log spiral scale over the index on the base. This has the advantage that the disk is also set at this number, and it can be used directly in operations involving multiplication or division.

An ln scale permits determination of powers of e from 0.23 to 2.3.

The Tyler's Fatal Flaws

The Tyler involves some clever applications of the log spiral. However, the device is not very practical. First, the basic one-cycle log scale is less precise than the larger circular scale that could have been installed on the same disk. Second, the A, T, S, and ln scales depend on identifying the intersection of two lines that are often almost parallel. In my attempt to use these scales, I found it very difficult to set and read these scales closer than two significant digits. Third, the array of often-intersecting curves made reading and settings near such intersections very confusing and unreliable. Finally, since the log spiral scales depend on distance from the center of the spiral, the disc must remain very precisely centered. One of the specimens that I examined had too much slack or "play" around the central rivet.

Conclusion

These two log spiral calculating devices are interesting and deserve to be recognized by slide rule collectors, but it easy to understand why these devices were not widely used and specimens are relatively rare.

Acknowledgements

The author is grateful to Bruce Reichelt for the opportunity to examine the Tyler in his collection. Alan Morris contributed the information about Captain Weems' office and called attention to the problem of the nearly parallel intersecting curves.