INSTRUCTIONS

FOR THE USE OF THE

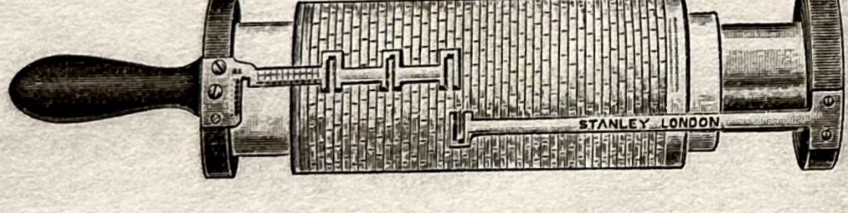
CO-ORDINATE SPIRAL SLIDE RULE

BY

H. O. BARNARD, A.C.H., F.R.A.S., &c.

Superintendent Trigonometrical Surveys,

Ceylon Survey Department.



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CHAPTER III.

CHAPTER IV.

PREFACE.

The use of Slide Rules has spread so largely in recent years, that it is scarcely necessary to dwell upon their many advantages. To every professional man who has to deal, even occasionally, with figures, the slide rule has come as a priceless boon, enabling him to perform the most complicated computations with a saving of nine-tenths of the time which ordinary methods would take, while the tax on the brain is reduced to the mere setting and reading of two indices. Even this, after only a few weeks' practice, becomes almost automatical, to an extent which can scarcely be realized by the novice; and, in addition, the mind is gradually and unconsciously trained in what may be called the philosophy of computation, seizing by instinct upon the shortest methods of work, while the order of magnitude and the approximate value of the result suggest themselves to the mind without effort.

Such are some of the advantages which may be secured by a few minutes' daily practice of the instrument during a few weeks: it will be readily conceded that even if months instead of weeks were required to obtain them, the time so spent would be laid out to the greatest advantage, since the economy of time and trouble is increased with every single computation performed throughout life.

The ordinary 10-inch slide rule, although extremely useful for many purposes, is not accurate enough for the requirements of the majority of professional men. Increase in accuracy can only be attained by increase in the length of its scales, when it speedily becomes unwieldy. This difficulty was overcome in 1650 by Milburne, who first thought of wrapping a logarithmic scale spirally over a cylinder; but the spiral slide rule was not evolved in a practical form before 1878, when Prof. Fuller, of Belfast, produced the admirable instrument which bears his name, and which is so favourably known to all engineers.

The use of Prof. Fuller's rule is, however, confined to arithmetical computations. The numerical solution of formulæ comprising trigonometrical functions can only be performed by extracting, with considerable loss of time, the values of these functions from a book of tables. To do so requires a certain effort of mind with its consequent risk of mistakes. This limitation has restricted its use in a considerable body of calculations, such, for example, as in the computation of the co-ordinates of surveys from the lengths and bearings of their lines, a method of plotting which is very largely used by Land Surveyors at present; in Astronomical computations; in Civil and Mechanical Engineering, &c.; the use of logarithms being preferred on the score of speed, although the degree of accuracy attained with Prof. Fuller's rule is amply sufficient in the large majority of cases.

The Co-ordinate Spiral Slide Rule has been designed to meet these requirements. Like Prof. Fuller's rule, upon which it is an improvement, it enables the user to perform, with speed and accuracy, arithmetical computations involving: Multiplication, Division, Proportion, Continuous Fractions, Powers, Roots, and Logarithms; but, in addition, the natural and logarithmic values of trigonometrical functions of any angle can be determined by inspection with the same accuracy as in numerical computation, while the products, quotients, etc., of these functions by lengths or numbers, integral or fractional, are obtained with equal ease, rapidity and precision. The scope of its operations will be gathered from the examples which are given to illustrate its use in the chapters which follow.

Although the Co-ordinate Spiral Rule, as all varieties of slide rules, is based primarily upon the theory of logarithms, a knowledge of that theory is by no means essential to its practical use. For this reason, a concise account of the theory and properties of logarithms and of the principles of construction of logarithmic scales has been relegated to the last chapter, where the reader will find the information necessary to a thorough understanding of the rule, expressed as simply as possible consistently with the subject.

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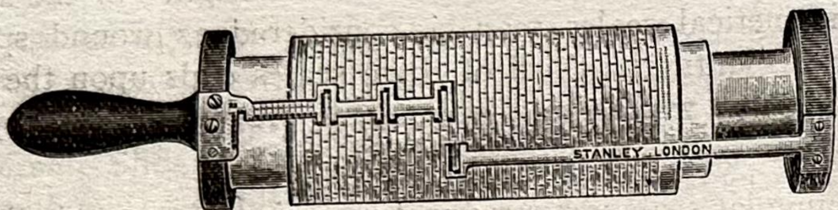


CHAPTER I.

DESCRIPTION.

THE Co-ordinate Spiral Rule is made up as follows: a plain cylinder *B* (fig. 1) is firmly mounted upon a wooden disc (*d*) to which the handle (*h*) is screwed. It is lined with cloth inside, forming a bearing for a brass tube *A*, which slides smoothly within it. Firmly fixed to this tube is a perforated wooden plate (*p*), which carries an index (*i*), called the "moveable index." Over the cylinder *B* slides an outer cylinder *C*, upon the surface of which scales are engraved. The inside of this cylinder is also lined with cloth to ensure a smooth and even motion. A brass index (*v*), the "vernier index," is attached by three screws (*a*, *b*, *c*,) to the disc *C*. It is clear that the index (*i*) can be placed in any position relatively to the vernier index (*v*), while the cylinder *C* may be moved up or down, or rotated over *B*, without disturbing the relative position of the two indices.

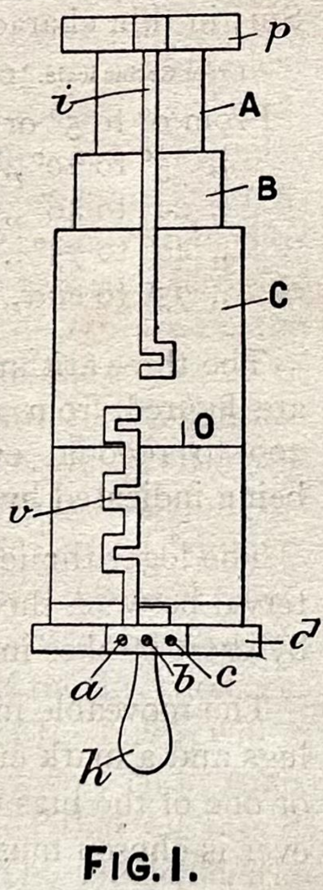


FIG. I.

The handle (*h*) can be unscrewed and placed within the brass tube *A*, thus conveniently shortening the instrument for packing or transport.

Scales.—The outer cylinder *C* bears five scales. The upper half is occupied by the scale of trigonometrical functions or "angular scale"; the lower half is taken up by three identical scales of numbers or "arithmetical scales" following each other, while the bottom of the cylinder is uniformly divided, forming the scale of logarithms. The angular scale joins the first of the arithmetical scales at a point marked O, Lat. Cos. / Dep. Sin.. From this point which will be called for convenience the "origin" of the scales, the arithmetical scales may be considered as wound spirally downwards until the third of these scales ends upon the logarithmic scale; while the angular scale proceeds in the opposite direction.

The angular scale is figured in two series of types, indicating the degrees and minutes. The heavy figures correspond to the scale of cosines, as indicated by the words lat: cos: being engraved in heavy characters; the thin figures complements to 90° of the angles on the cosine scale will evidently represent the Sines of these complements and are indicated by the words Dep: Sin: in thin characters. The angular scale is divided as follows:

	Lat or Cosine scale.		Dep. or Sin. scale.		
From	0° to 5°	or	90° to 85°—	into	single degrees;
,,	5° to 20°	,,	85° to 70°—	,,	half degrees or 30 minutes;
,,	20° to 40°	,,	70° to 50°—	,,	ten minutes;
,,	40° to 75°	,,	50° to 15°—	,,	five minutes;
,,	75° to end	,,	15° to end—	,,	single minutes.

The three arithmetical scales are identical with each other and are figured, from 100 to 200, at every even division: and from 200 to 1000 at every fifth division, the intervening numbers being indicated by subdivision lines.

The logarithmic scale is figured at every $\frac{1}{100}$ and $\frac{5}{1000}$, the interval between them being divided by long strokes into $\frac{1}{1000}$ and by short strokes into $\frac{5}{10,000}$.

The moveable index (*i*) is a plain brass bar, which carries two lugs and a mark engraved upon the lower one. Either the mark, or one of the lugs may be used at will as the pointer; but whichever is chosen must be used throughout one set of computations.

The vernier index (*v*) consists of a stiff bar carrying three pairs of lugs, the lower one of each pair bearing a mark engraved upon it. Below these lugs, the bar is divided into ten spaces, figured 0 to 9. The lower part of the index is formed into a vernier to subdivide the logarithmic scale, and the whole is firmly fixed to the wooden base by two screws with a large screw head between them.

PREPARATION TO THE USE OF THE RULE.

To use an instrument so that no conscious effort of mind is required to perform with it the work it is designed to carry out, is an art which can only be acquired by constant practice. The Co-ordinate Slide Rule is no exception to this law; and the following hints, the result of long experience, will be of considerable help to the beginner in mastering the instrument.

In the first place a few minutes must be spent daily in noting carefully the *values* of the different divisions on the rule, and an effort must be made to remember the relative position of some of the principal numbers on the arithmetical and angular scales along say three lines parallel to the indices at each third of the circumference. Thus, for instance, a glance at the arithmetical scales will show that the numbers 100, 200, 400, 800, 1000, multiples of 100 are more or less in line at one portion of the cylinder *C*; that the numbers 110, 220, 440, 880, multiples of 110 and 175, 350, 700, multiples of 175 are also in line after turning the cylinder by about $\frac{1}{3}$ of its circumference; and that 120, 240, 480, 960, multiples of 120 and 150, 300, 600, multiples of 150 are on the same portion of the cylinder, $\frac{1}{3}$ of the circumference to the left of the origin or $\frac{2}{3}$ of it towards the right. Whenever an index has to be set to a given number, a great deal of time will be ultimately saved by first ascertaining whereabouts it may be found on the cylinder, before shifting either the moveable index or the cylinder; and if the three positions of the series of numbers are remembered, even roughly, a glance at the portion of the scale immediately in front of the eye will enable the user to know the direction and amount of motion required, of the cylinder or of the index, which is necessary to accomplish his object. After a very short time, the eye and hand become so thoroughly trained that all this is done instinctively and with great rapidity. The same remarks hold good for the angular scale.

Next the beginner should learn to estimate fractional parts of one division, such as thirds, quarters and tenths. It may appear preposterous to those who do not habitually use graduated scales to expect that the eye is able to discriminate between such small quantities; but a very few trials will convince the reader that it is not only possible, but easy to do so. He should draw two short lines parallel to each other about $\frac{1}{2}$ inch apart, and mark with a pencil haphazard any estimated fraction and check the result by careful measurement with a finely graduated scale. He will then be agreeably surprised to find his estimate correct within $\frac{1}{10}$ of the original interval between the lines. Very little practice will enable him to estimate with certainty $\frac{1}{20}$ of the interval. He should next diminish the distance between the parallel lines, and finally test his training upon large and small intervals chosen at random.

ADJUSTMENT OF THE RULE AND ITS PRESERVATION, ETC.

The only adjustment which must be attended to before using the rule, is that all the lugs upon the vernier index, or the three marks upon them, should be strictly parallel to the axis of the rule. This happens when the three marks read exactly the same number upon each of the arithmetical scales. The adjustment is made as follows: the screw on the left of the vernier index, which secures it to the base plate, is loosened by a quarter or half a turn. The large screw head, which is really an encentric, is slightly turned backwards or forwards, until the three marks read the same number on the scales. The left hand screw is then tightened. This adjustment is made once for all and is not liable to be disturbed, unless the rule has been subjected to some violent jar. It should however be tested regularly about once a month or so.

While in use the rule should be stood on the table handle upwards in the intervals of computation; but at all other times, it should be put back into its box, and carefully guarded against accidents. If through damp, or long disuse, the cylinders move stiffly or jerkily, a *little* French chalk applied upon the cloth linings as a lubricant will restore its ordinary smoothness of movement. The rule must never be exposed to the sun.

The best way of holding the rule in working, is to seize firmly the handle with the fingers of the right hand, the ball of the thumb resting upon the graduated portion of the vernier index, slightly above the level of the vernier. A slight pressure of the thumb will then depress the index and force it into contact with the scales when any parallax in reading is avoided. The left hand grasps the wooden plate bearing the moveable index, and twists or slides it into the position required; it also works the cylinder *C*, the right hand remaining always in the position described.

CHAPTER II.

USE OF THE RULE—ARITHMETICAL SCALES.

Multiplication: To multiply two numbers together, turn cylinder *B* until one of the pointers (marks or lugs) on the vernier index reads one of the given numbers. The pointer of the moveable index is made to coincide with the origin or with the 100 mark on any of the scales, and finally the cylinder *C* is moved so that the second number is brought up to the moveable index, when the result is read by one of the pointers of the vernier index.

Example (1): 325 × 27.

(*a*) The vernier index is set to 325 by revolving cylinder *C*
(*b*) The moveable index is set to the origin
(*c*) The cylinder *C* is moved until the moveable index reads 270, when the vernier index reads the result 8775.

Continued Multiplication: When three or more factors have to be multiplied together, proceed as follows:

(*a*) Set the vernier index to the first factor
(*b*) Set the moveable index to the origin
(*c*) Move the cylinder *C* until the moveable index reads the second factor
(so far the procedure is identical with that described above) then,
(*d*) Set the moveable index to the origin
(*e*) Move the cylinder to make the moveable index read the third factor.
The result is read off by the vernier index: if there are more than three factors, repeat as follows:
(*f*) Set the moveable index to the origin
(*g*) Move the cylinder to make the moveable index read the fourth factor and so on.

Example (2): 10.7 × 3.13 × 2.75 × 5.10 = 469.7.

(*a*) Vernier index to 107
(*b*) Moveable index to origin (100)
(*c*) Turn cylinder to make moveable index read 313
(*d*) Moveable index to origin
(*e*) Turn cylinder to make moveable index read 275
(*f*) Moveable index to origin
(*g*) Turn cylinder to make moveable index read 510
(*h*) Read result on vernier index = 4697.

Important Remark: It will be noticed that the *vernier index* is read only twice: first in setting it to the first factor, and second reading the result, all intermediate operations being performed by shifting the *moveable index* and the *cylinder alternately.*

Constant Factor: It frequently happens that a constant quantity has to be multiplied by a series of numbers; in that case proceed as follows:

(*a*) Set the vernier index to the constant factor

(*b*) Moveable index to the origin

(*c*) Turn cylinder to make moveable index read each of the different numbers in succession.

This is merely the same rule as for simple multiplication except that it is not necessary to repeat (*a*) and (*b*).

Example (3): Find the circumference of circles whose diameters are 1.25, 3.07, 4.59. The formula for finding the circumference is $C = \pi d$, where C is the circumference, d the diameter, $\pi = 3.142$ the proportion of the circumference to the diameter. Hence we have to find the value of

$$3.142 \times 1.25 = 3.927$$
$$3.142 \times 3.07 = 9.645$$
$$3.142 \times 4.59 = 14.420$$

(*a*) Set vernier index to the value of $\pi = 3.142$

(*b*) Moveable index to origin

(*c*) Turn cylinder to make moveable index read 125

(*d*) Read first result at vernier index = 3927

(*e*) Turn cylinder to make moveable index read 307

(*f*) Read second result at vernier index = 9645

(*g*) Turn cylinder to make moveable index read 459

(*h*) Read third result at vernier index = 14420.

Similarly when a number of constant factors have to be multiplied by a series of numbers, the value of the product of the constant factors is first obtained, and treated as the value of π in the above example. There is, however, no necessity to *read* that value unless it is desired to do so. Thus, suppose the cubic contents of a number of prisms of the same section but of different lengths is required, such as

Example (4):
$$4.13 \times 2.05 \times 7.23 = 61.22$$
$$4.13 \times 2.05 \times 9.15 = 77.47$$
$$4.13 \times 2.05 \times 1.75 = 14.82 \quad \text{etc.}$$

As before
(*a*) Set the vernier index to 413
(*b*) Moveable index to origin
(*c*) Turn cylinder to make moveable index read 205
(*d*) Moveable index to origin
(*e*) Turn cylinder to make moveable index read 723
(*f*) Read first result at vernier index = 6122
(*g*) Turn cylinder to make moveable index read 915
(*h*) Read second result at vernier index = 7747 and so on.

Division: To divide a given number by another, proceed as follows:
(*a*) Set the vernier index to the given number
(*b*) Set moveable index to the divisor
(*c*) Turn cylinder to make the moveable index read 100
(*d*) Read the result at the vernier index

Example (5):
$$\frac{29.56}{9.41} = 3.142$$
(*a*) Set vernier index to 2956
(*b*) Moveable index to 941
(*c*) Make moveable index read 100
(*d*) Read result at vernier index 3142

Continued Fractions: To multiply together a series of fractions, proceed as in continued multiplication, except that the cylinder is turned so as to make the moveable index read the next numerator instead of the origin, or 100; thus:
(*a*) Set the vernier index to the first numerator
(*b*) Set the moveable index to the first denominator
(*c*) Turn cylinder to make moveable index read second numerator
(*d*) Set moveable index to the second denominator
(*e*) Turn cylinder to make moveable index read third numerator
(*f*) Read the result at the vernier index

Example (6):
$$\frac{2.52 \times 3.67 \times 9.23}{1.09 \times 7.64} = 10.25$$
(*a*) Set vernier index to 252
(*b*) Set moveable index to 109
(*c*) Turn cylinder to make moveable index read 367
(*d*) Set moveable index to 764
(*e*) Turn cylinder to make moveable index read 923
(*f*) Read result at vernier index 1025.

When the number of terms in the numerator exceeds that of the denominator, or *vice versa*, write mentally 1.00 for each of the terms in excess in order to make the number of terms in the numerator always one more than in the denominator, and proceed as above; thus:

Example (7): $\frac{3.59 \times 1.75 \times 2.93 \times 5.04}{17.59}$ would be written mentally as $\frac{3.59 \times 1.75 \times 2.93 \times 5.04}{17.59 \times 1.00 \times 1.00}$; and the value of this expression would be computed as follows:

(*a*) Set vernier index to 359
(*b*) Set moveable index to 1759
(*c*) Turn cylinder to make moveable index read 175
(*d*) Set moveable index to 100
(*e*) Turn cylinder to make moveable index read 293
(*f*) Set moveable index to 100
(*g*) Turn cylinder to make moveable index read 504
(*h*) Read result at vernier index = 5275

The result being 5.275.

Example (8): Again the expression $\frac{2.95}{1.07 \times 1.13 \times 2.67}$ would be written mentally as: $\frac{2.95 \times 1.00 \times 1.00 \times 1.00}{1.07 \times 1.13 \times 2.67}$ and would be computed in exactly the same way; thus:

(*a*) Set the vernier index to 295
(*b*) Set moveable index to 107
(*c*) Turn cylinder to make moveable index read 100
(*d*) Set moveable index to 113
(*e*) Turn cylinder to make moveable index read 100
(*f*) Set moveable index to 267
(*g*) Turn cylinder to make moveable index read 100
(*h*) Read result at vernier index = 9138

The result being 0.9138

It will be seen, that, as in multiplication, the vernier is never read more than twice: first to set the first factor, and lastly to read the result, the whole work being performed by the moveable index and cylinder.

Important Remark: A study of the operations given in detail above, both for multiplication and division will show that they reduce themselves to the following:

(1) The vernier index is only used for setting the *first factor* and to read the *result*
(2) Moving the cylinder up to the moveable index *multiplies*
(3) Setting the moveable index *divides*.

The reader is strongly recommended to make examples for himself and to practice constantly the above methods, which constitute the *key to the use of the Rule* until they are performed automatically. This should not take more than a few days.

In what follows, it will be assumed, that the necessary proficiency has been attained; and, to avoid repetition, the word "*multiply*" will be taken to indicate always that the cylinder is moved to make the moveable index read the given number; and the word "*divide*" that the moveable index is set to the given number.

Position of the Decimal Point: The numbers on the arithmetical scales are figured from 100 to 1000: for reasons which are given in the chapter on logarithms, they give no indication of the order of magnitude, any magnitude being assigned to the numbers at will. Thus, the number 935 may be taken to mean 935, 93.5, 9.35, 0.935, etc. It is, however, of great importance to know what the result obtained really means. Of the many rules which may be devised to determine the magnitude of the result given by the rule, only two can be recommended as short and practical. The easiest and most successful whenever the expressions to be computed are not too complicated, is to compute them mentally, taking their terms as the nearest whole numbers. Thus in the examples already given, we have:

Example (1): $325 \times 27 = 8775$. This would be taken roughly as 300×30 which gives 9000: the product is therefore 8775.

Example (2): $10.7 \times 3.13 \times 2.75 \times 5.10 = 469.7$
Compute mentally $10 \times 3 \times 3 \times 5$ which is 450: the result is therefore read as 469.7.

Example (6): $\dfrac{2.52 \times 3.67 \times 9.23}{1.09 \times 7.64} = 10.25$
Compute mentally $\dfrac{2 \times 4 \times 9}{1 \times 8}$ which is 9: the result is therefore 10.25.

Example (7): $\dfrac{3.59 \times 1.75 \times 2.93 \times 5.04}{17.59} = 5.275$
Compute mentally $\dfrac{4 \times 2 \times 3 \times 5}{20} = 6$: the result is therefore 5.275.

Example (8): $\frac{2.95}{1.07 \times 1.13 \times 2.67} = 0.9138$

Compute mentally $\frac{3}{1 \times 1 \times 3} = 1$: the result is therefore 0.9138.

The second rule consists in converting mentally all the terms of an expression into units and decimals multiplied by powers of 10. Thus, Example (1) would be written mentally as:

$3.25 \times 10^2 \times 2.7 \times 10^1$, the result being 8.775 by 10^{2+1} or 8.775×1000, that is 8775.

This rule is extremely useful when numbers have to be multiplied or divided by small decimal fractions. Suppose, for instance, that the expression to be computed is

Example (9): $\frac{325 \times .075 \times 2.13}{.008 \times 125 \times 1.92}$ to obtain the magnitude of the result, would take some consideration and consequently some time by the first rule; whereas if we imagine it to be written $\frac{3.27 \times 10^2 \times 7.5 \times 10^{-2} \times 2.13}{8.0 \times 10^{-3} \times 1.25 \times 10^2 \times 1.92 \times 10^{-1}}$ we see that the expression becomes equivalent to

$$\frac{3.27 \times 7.5 \times 2.13 \times 10^{2-2}}{8.0 \times 1.25 \times 1.92 \times 10^{-3+2-1}}$$

$$\text{or} \quad \frac{3.27 \times 7.5 \times 2.13}{8.0 \times 1.25 \times 1.92} \times \frac{10^0}{10^{-2}}$$

the result being $2.720 \times 10^2 = 272.0$.

It will be noticed that, in the above example, the result is read on the same arithmetical scale. When the above rule is applied, that is when all the terms of the expression are supposed to be units, the scale upon which the final result is read must be carefully considered. If we call the arithmetical scale beginning at the origin the first scale, the next following, the second, and the last scale the third; then if all the terms of the expression are set on the first scale and the result is read on the second, it is to be read as ten times greater; and, if on the third, as a hundred times greater than if it had been read on the same scale, irrespective of the powers of 10. Thus, for instance, suppose the expression to be:

Example (10): $\frac{3.33 \times .082 \times 912}{.006 \times 22.3}$

Applying the rule we imagine it to be written as

$$\frac{3.33 \times 8.2 \times 9.12 \times 10^{-2+2}}{6 \times 2.23 \times 10^{1-3}}$$

$$\text{or } \frac{3.33 \times 8.2 \times 9.12}{6 \times 2.23} \times 10^2$$

Performing the computation on the *first* scale, we read the result 1861 with the same lug on the *second* scale. As all the terms are units, the result will be ten times as great or $18.61 \times 10^2 = 1861$. Had it been read on the first scale, it would have been 1.861×10^2 in accordance with the rule.

Similarly, if we call the marks or lugs of the vernier index first, second and third in the same descending order as the scales, the above applies only when the same lug of the index is used. If, using the first lug, the result is read upon the same scale, but with the second lug, the order of magnitude is $\frac{1}{10}$ of what it would otherwise have been, and so on.

Thus, for instance, take the following expression:

Example (11): $\frac{15.93 \times .003}{7.59 \times .924}$ applying the rule, the expression is reduced to $\frac{1.593 \times 3.0}{7.59 \times 9.24}$ performing the computation on the instrument with the *first* lug, and using the *first* scale throughout, we find that the result 6814 is read with the *second* lug on the *first* scale. It therefore is 0.6814×10^{-1} or 0.06814.

The above rule may be summarised as follows:

The expression to be computed being in units,

The result read with the first lug on the 1st scale " " " second " " 2nd " " " " third " " 3rd "	is in units.
" " " first lug on the 2nd scale " " " second " " 3rd "	is in ten units.
" " " first lug on the 3rd scale	is in hundred units.
" " " second lug on the 1st scale " " " third " " 2nd "	is in tenths of units.
" " " third lug on the 1st scale	is in hundredths of units.

Reciprocals: The reciprocal of a given number is merely unity divided by that number. They are attained by setting the vernier index to 100, the moveable index to the given number, and turning the cylinder until the moveable index reads the origin. The number then read by the vernier index is the decimal fraction representing the reciprocal of the given number. The rules already given for the position of the decimal point must be applied as in the case of ordinary division.

Proportion: (i.) To find a *fourth* proportional in the expression $\frac{a}{b}=\frac{c}{x}$ is merely to compute a fraction, since the expression may be put in the form $x=\frac{b \times c}{a}$. It is, however, simpler to

(*a*) Set vernier index to number represented by *c*
(*b*) Set moveable index to that represented by *a*
(*c*) Multiply (*) by that represented by *b*
(*d*) Read the result at the vernier index

Example (12): One pound sterling being worth 25 francs 20 centimes, what is the equivalent of 16 shillings in francs?
(*a*) Set the vernier index to 2520: it will then represent francs
(*b*) Set the moveable index to 100: it will then represent pounds
(*c*) Multiply by 8: (16 shillings = 0.8 £)
(*d*) Read the result 20.16 francs at the vernier index.

All problems in simple or double rule of three are solved in exactly the same way.

(ii.) To find a *mean* proportional in the expression $\frac{a}{x}=\frac{x}{b}$, or $x^2 = ab$, it is necessary to extract the square root in the following manner.

Square Root: When any number is multiplied or divided by 100, its square root is multiplied or divided by 10: the square root of a given number will therefore be some number between 0 and 10 multiplied or divided by an even power of 10.

To extract the square root of a number, divide that number in groups of two figures, on each side of the decimal point. The number of significant figures in the root will then be the number of groups of two figures in the integral part.

* See *Important Remark*, page 15.

The squares of whole numbers between 0 and 10 are easily remembered: they are

No.	0	1	2	3	4	5	6	7	8	9	10
Square	0	1	4	9	16	25	36	49	64	81	100

Having divided the number into groups of two figures, divide or multiply it by an even power of ten so that it will lie between 0 and 100. From the above table, make a rough guess at the root and proceed as follows:

(*a*) Set the vernier index to the number guessed
(*b*) Multiply it by itself
(*c*) If the product, as is probable, is not the given number, turn the cylinder to make the vernier index read that number
(*d*) Read the resulting number at the moveable index
(*e*) Take the mean between that number and the number guessed. This will be the square root required, if the difference between the two is not too great. If it is, take the mean as the number guessed and repeat the process, which will then give the root accurately.

Example (13): Extract the root of 817.96. This is equivalent to 8.1796 × 100.

The nearest square in the table is 9, whose square root is 3. But as 8.1796 is smaller than 9, we should naturally guess the square root to be smaller than 3, about 2.90 for instance.

To illustrate the method, however, let the number guessed be 3.00.

(*a*) Set the vernier index to 300
(*b*) Multiply it by itself: the result is 9
(*c*) Move the cylinder so that the vernier index reads 8.1796
(*d*) The moveable index now reads 2.726
(*e*) The mean of 3.000 and 2.726 is 2.863.

Repeating the process with 2.863 instead of 3.000 we find 8.197 as its square; making the vernier index read 8.1796, the moveable index reads 2.857 the mean of the two numbers being 2.860 which is the correct square root of 8.1796. The square root of 817.96 will of course be 28.60, since 817.96 is equivalent to 8.1796 × 100.

Example (14): Extract the square root of 8855.

This is equivalent to 88.55 × 100 and the root will have two figures before the decimal point.

The nearest square to 88 is 81, the root of which is 9. As a first guess, try 9.30 as the root.

(*a*) Set the vernier index to 9.30
(*b*) Multiply it by itself: the result is 8650
(*c*) When the vernier index is set to 8855, the moveable index reads 9.519. The mean of 9.300 and 9.519 is 9.410; and as there are two figures in the root before the decimal point, the square root of 8855 is 94.10.

Cube Root: The method of finding the cube root of a given number is very similar to the one by which the square root is obtained. It is based upon the following considerations.

When any number is multiplied or divided by 1000, its cube root is multiplied or divided by 10. The cube root of a given number will therefore be some number between 0 and 10, multiplied or divided by some power of 10.

If the given number be divided into groups of three figures on each side of the decimal point, the cube root will have as many figures before the decimal point or as many zeros after it as there are groups of three figures or of three zeros in the number.

The following are the cubes of whole numbers between 0 and 10.

No.	0	1	2	3	4	5	6	7	8	9	10
Cube	0	1	8	27	64	125	216	343	512	729	1000

Divide or multiply the number by 10^3 or 10^6, etc., so that it will lie between 0 and 1000. From the above table make a rough guess at the value of the cube root, and proceed as follows:

(*a*) Set the vernier index to the number guessed
(*b*) Cube it, *i.e.*, multiply it by itself three times
(*c*) Make the vernier index read the given number
(*d*) Read the new reading of the moveable index and add to or subtract one-third of the difference from the number guessed. If the difference between the number guessed and that read is not too great, the result will be the cube root sought.

Example (15): Find the cube root of 2686: this is equivalent to 2.686 × 1000.

From the table above, the cube root is evidently slightly greater than 1 and much less than 2. Try 1.25.

(*a*) Set the vernier index to 1250
(*b*) Cube it (the result, which need not be read, is 1953)
(*c*) Make the vernier index read 2686
(*d*) The moveable index reads 1719.

The difference between these numbers is 0.469, one third of which, or 0.156 is to be added to 1.250 (the number guessed) thus making 1.406. The difference between 1.250 and 1.719 is however too great. Repeating the operation with 1.406 instead of 1.250 we have 2779: setting the vernier index to 2686, the moveable index reads 1.359: the difference is now .047, one third of which ·016 is to be subtracted from 1.406, the result being 1.390; and since the root should have two figures, it is clearly 13.90.

Example (16): Find the cube root of 0.001602.

This is the same as 1.602 divided by 1000; and the first figure of the root will come immediately after the decimal point. Try 1.10 and proceed as before: the moveable index reads 1.324: the difference 0.224, one third of which .075, added to 1.100 gives 1.175. Repeat the operation; the moveable index gives 1.160: the difference is .015, one third of which .005, subtracted from 1.175 gives 1.170, the correct cube root. This root must, however, be divided by 10 since the number was divided by 1000. Hence the correct cube root of 0.001602 is 0.1170.

Powers of Numbers: When the index of the power to which the given number must be raised does not exceed three or even four, the quickest method is to multiply the number by itself; when it exceeds that limit, and also when the index of the root required is greater than three, recourse must be had to logarithms, which are read on the Rule as follows:

Logarithms: At the bottom of cylinder *C* is a scale graduated uniformly, and read more closely by the vernier at the base of the vernier index. The lower portion of the latter is divided on the bar and figured from 0 to 9. These divisions cover the third or lower arithmetical scale from 100 to 1000; and the width of each division is the height between two spirals.

Only the *mantissa*, or decimal fraction of the logarithm is given by the rule, the *characteristic* being supplied by the user.

To find the logarithm of a number, first write down the characteristic which is equal to the number of figures in the number before the decimal point, less one. The mantissa is then obtained by turning the cylinder, while it rests upon the wooden plate at the bottom, until the divided perpendicular edge of the vernier index is in line with the division on the third arithmetical scale representing the number. The first place of decimals will be given by the figure engraved in the division within which the number appears, the remaining decimal places being read by the scale and vernier. Thus:

Example (17): To find the logarithm of 272.0: this number has three significant figures: its characteristic is therefore 2. Bring the edge of the graduated bar over 272; the division next to it on the bar is 4, and the vernier reads 3455. Hence the log. of 272 = 2.43455. Again:

Example (18): To find the logarithm of 40.57: here the characteristic is 1, the number on the bar when brought over 4057 is 6, while the vernier reads 0822: the log. of 40.57 is therefore 1.60822.

In the case of the decimal fractions, the characteristic is negative and is equal to the number of zeros before the first significant figure, plus one, the mantissa being obtained in exactly the same manner as described. Thus:

Example (19): to find the logarithm of 0.000912: the characteristic is $\bar{4}$ and the log. is $\bar{4}.96000$. The mantissa is always positive.

To find the number corresponding to a given logarithm, no notice is taken at first of the characteristic. The vernier is made to read the second, third, fourth and fifth places of decimals and the number required is read opposite the division on the bar corresponding to the first decimal place.

Example (20): To find the number whose logarithm is 1.76745: make the vernier read 06745; and opposite the division 7 on the bar, the number read is 5854. The characteristic being 1, there are two figures before the decimal point. The number required is therefore 58.54.

As previously stated, logarithms need only be found when powers or roots of a higher index than the third are required as in the following examples:

Example (21): Find the value of £567 at 4% compound interest after 15 years.

The formula is $M = P\,(1 + r)^n$ where M is the amount, P the principal, r the rate and n the number of years.

Substituting the given values for the symbols $M = 567\,(1 + .04)^{15}$ Find the logarithm of 1.04, which is 0.01705; multiply this by 15, which gives 0.2555; find the number corresponding to this logarithm, which is 1.801; multiply this by 567, the result is £1021.

Example (22): What is the rate of compound interest at which a capital of £235 becomes £500 in 12 years?

Writing the formula as $(1 + r) = \sqrt[12]{\frac{500}{235}}$

The value of the fraction $\frac{500}{235}$ is 2.128, the logarithm of which is 0.32790. Dividing this by 12 we get 0.02744 and the number corresponding to it is 1.065,

Hence $r = .065$ or $6\frac{1}{2}\%$.

Example (23): In how many years will a capital of £210 become £500 at $5\frac{1}{2}\%$ compound interest?

Transforming the formula, we have

$$n \log.\,(1 + r) = \log.\,\frac{M}{P} \text{ and } n = \frac{\log.\,\frac{M}{P}}{\log.\,(1 + r)}$$

The fraction $\frac{M}{P} = \frac{500}{210} = 2.381$, and its logarithm is 0.37675.

The term $(1 + r) = 1.055$ and its logarithm is 0.02325

$$n = \frac{0.37675}{0.02325} = 16.21 \text{ years.}$$

CHAPTER III.

ANGULAR SCALE.

The reader is now supposed to be completely at home in the several operations which can be performed on the arithmetical scales. As stated previously, the angular scale is a scale of cosines (in heavy type) beginning with 0° at the origin and ending at the top of the cylinder at 89° 56′ 34″. The same scale, as explained already, forms at the same time a scale of sines (in thin type) beginning at 0° 3′ 26″ at the top and ending at 90° at the origin.

The value of the natural sine of 0° 3′ 26″ or of the cosine of 89° 56′ 34″ is 0.001, while the natural sine of 90° or the cosine of 0° is 1.000: so that the rule can deal directly with angles whose natural sines lie between .001 and 1.000 and whose natural cosines lie between 1.000 and .001.

When it is desired to obtain values of these functions for angles smaller than 0° 3′ 26″ or greater than 89° 56′ 34″, they may be computed as follows:

The arc, sine, and tangent of very small angles or the cosine and co-tangent of their complements are all equal, and their value is obtained by multiplying the angle expressed in minutes by $\frac{\pi}{180 \times 60}$ or by 0.0002909, or the angle expressed in seconds by $\frac{\pi}{180 \times 60 \times 60}$, or by 0.000004848; thus:

Example (24): The arc, sine or tan. of 2′54 or the cosine or co-tangent of 89° 57′ 6″ is equal to 2′.90 × 0.0002909 or .0008436, which is the same as 174″ × .000004848.

Reciprocally to find the angle whose sine, arc or tangent is smaller than 0.001, multiply by 3437.75 to obtain its value in minutes, or by 206265 if it is required in seconds. These numbers are the reciprocals of .0002909 and .000004848 respectively.

The functions of small angles may, however, be found on the rule by finding their values for angles 60 times as great, and dividing the result by 60. Thus, as in the preceding example, to find the sine arc or tangent of 2′ 54″, this angle multiplied by

60 becomes 2° 54″; and its sine found on the rule as explained below is 0.05059, which, divided by 60, gives .0008436 as before.

Natural Sines and Cosines: To find the value of the natural sine of angles smaller than 0° 3′ 26″ or of the natural cosine of their complements:

(*a*) Set the vernier index to 100 at the end of the *first* arithmetical scale, if the angle (*scale of sines*) is between 90° and 5° 45′; at the end of the *second* arithmetical scale if it is between 5° 45′ and 0° 43′; and at the end of the *third* arithmetical scale, if it is between 0° 3′ and 0° 43′ (or the complements of these angles on the scale of cosines)

(*b*) Set the moveable index to the origin

(*c*) Turn the cylinder to make the moveable index read the **division representing the angle whose sine or cosine is sought**

(*d*) Read the vernier index which will give the required value.

The decimal point is placed in accordance with the following rules:

If the value is read simultaneously at the

first lug on the first scale second ,, ,, ,, second ,, third ,, ,, ,, third ,,	The value of the function is in tenths of unity, or the first figure comes next to the decimal point.

If it is read simultaneously at the

first lug on the second scale second ,, ,, ,, third ,,	The value of the function is in hundredths of unity, or there is one zero after the decimal point.

If it is read by the first lug on the third scale	The value of the function is in thousandths of unity, or there are two zeros after the decimal point.

Example (25): Find the sine of 38° 15′.

As this angle lies between 5° 45′ and 90°

(*a*) Set the first lug of the vernier index to the end of the *first* scale

(*b*) Set the moveable index to the origin

(*c*) Turn cylinder until the moveable index reads 38° 15′ in thin type

(*d*) The vernier index reads 6191, which in accordance with the rule just laid down is 0.6191.

This is also the cosine of 51° 45′ (heavy type).

Example (26): Find the cosine of 86° 12′.

As this angle lies between 89° 17′ and 84° 15′

(*a*) Set the first lug of the vernier index to the end of the *second* scale

(*b*) Set moveable index to the origin

(*c*) Turn cylinder until the moveable index reads 86° 12′

(*d*) The vernier index reads 6627, which, in accordance with the rule, is 0.06627.

To find the Angle corresponding to a given Sine or Cosine: This is, of course, merely the inverse of the preceding operation.

(*a*) Set the first lug of the vernier index to the end of the first scale, if the given function is in the first place of decimals; to the end of the second scale, if it is in the second place; and to the end of the third scale, if it is in the third place of decimals.

(*b*) Set the moveable index to the given number, always on the scale at the end of which the first lug of the vernier index is placed

(*c*) Turn the cylinder until the first lug of the vernier index points to the origin

(*d*) Read the angle shown by the moveable index: the thin figures if the function is a sine, the heavy figures if it is a cosine.

Example (27): What is the angle whose cosine is 0 3856?

As the number is in the first *place* of decimals

(*a*) Set the first lug of the vernier index to the end of the *first* scale

(*b*) Set the moveable index to 3856 on the first scale

(*c*) Make the first lug of the vernier index read the origin

(*d*) The moveable index reads 67° 19′ heavy type as the function is a cosine

Example (28): What is the angle whose sine is 0.07759?

The number being in the second place of decimals,

(*a*) Set the first lug of the vernier index to the end of the *second* scale

(*b*) Set the moveable index to 7759 on the first scale

(*c*) Make the first lug of the vernier index read the origin

(*d*) The moveable index reads 4° 27′ thin type as the function is a sine

Natural Co-secants and Secants are obtained as follows:

(*a*) Set the first lug of the vernier index to the origin

(*b*) Set the moveable index to the sine or cosine of the given angle, that is, to the thin or thick figures representing the angle on the angular scale

(*c*) Turn the cylinder until the moveable index points to the origin

(*d*) Read the first lug of the vernier index.

The order of magnitude is determined by the rules already given (*page* 18), that is, in other words:

If the first lug is read on the first scale, there is one figure before the decimal point;

If it is read on the second scale, there are two figures before the decimal point;

If it is read on the third scale, there are three figures before the decimal point.

Example (29): Find the co-secant of 14° 3′.

(*a*) Set the first lug of the vernier index to the origin

(*b*) Set the moveable index to 14° 3′ thin type, as the co-secant is the inverse of the sine

(*c*) Turn the cylinder until the moveable index points to the origin

(*d*) The first lug of the vernier index reads 4119 on the first scale. The co-secant of 14° 3′ is therefore 4.119.

Example (30): Find the secant of 89° 16′ 30″.

(*a*) Set the first lug of the vernier index to the origin

(*b*) Set the moveable index to 89° 16′ 30″ heavy type as the secant is the inverse of the cosine

(*c*) Turn the cylinder until the moveable index points to the origin

(*d*) The first lug of the vernier index reads 7904 on the second scale. The secant of 89° 16′ 30″ is therefore 79.04.

To find the angle from its secant or co-secant, the above operations are inverted. Take, for instance, the values of the last example:

(*a*) The first lug of the vernier index is set to 7904 on the second scale

(*b*) The moveable index is set to the origin

(*c*) The cylinder is turned until the first lug of the vernier index points to the origin

(*d*) The angle 89° 16′ 30″ is read at the moveable index.

MULTIPLICATION AND DIVISION OF TRIGONOMETRICAL FUNCTIONS.

We now come to one of the principal advantages of the Coordinate Slide Rule, viz., the possibility of operating upon trigonometrical functions directly on the angular scale, in the same way as with numbers upon the arithmetical scales, without the necessity of finding their numerical values. The first example of this property is in obtaining the values of

Natural Tangents and Co-tangents: The tangent of an angle is the ratio which the sine of that angle bears to its cosine; the co-tangent of an angle is the ratio of its cosine to its sine. These two relations are expressed by the formulæ,

$$\tan. A = \frac{\sin. A}{\cos. A}; \qquad \cot. A = \frac{\cos. A}{\sin. A}$$

The method of procedure is identical with that of division on the arithmetical scale, viz.:

(*a*) Set the vernier index to the sine of the given angle (thin type)
(*b*) Set the moveable index to the cosine of the angle (heavy type)
(*c*) Turn the cylinder so as to make the moveable index read 100
(*d*) The vernier index reading gives the value of the natural tangent of the angle.

When the co-tangent of the angle is required at the same time as the tangent, having found the tangent as above:

(*e*) Turn the cylinder so that the vernier index reads 100
(*f*) The moveable index will read the co-tangent.

The rules for the position of the decimal point are as stated previously, that is:

When one index is set to the end of the first arithmetical scale, and the other reads upon it, the first significant figure is immediately after the decimal point.

When one index is set to the end of the second scale, and the other reads on the first scale, there is one *zero* after the decimal point.

When one index is set to the end of the third scale, and the other reads on the first scale, there are two *zeros* after the decimal point.

When one index is set to the origin and the other reads on the first scale, there is one figure before the decimal point; if, on the second scale, there are two figures; if, on the third, three figures before the decimal point.

Example (31): Find the tangent and co-tangent of 33° 23′.

(*a*) Set the vernier index to the sine of 33° 23′ (thin type)
(*b*) Set the moveabie index to the cosine of 33° 23′ (heavy type)
(*c*) Turn the cylinder so that the moveable index reads 100 at the end of the first arithmetical scale
(*d*) The vernier index reads 0.6590, which is the natural tangent sought.
Without displacing the indices,
(*e*) Turn the cylinder to make the vernier index read the origin
(*f*) The moveable index reads 1.5175, the co-tangent required.

The co-tangent may, of course, be obtained independently by setting the vernier index to the cosine (heavy type) and the moveable index to the sine (thin type), and making the latter read the origin, when the co-tangent will be read at the vernier index.

The vernier index is too short to allow it to be set to the sine of angles smaller than 30°. In that case the indices are reversed. the only point to remember is that the index which has been set to the *denominator* (cosine for the tangent or sine for the co-tangent) must be made to read the origin or the end of an arithmetical scale *in all cases*. Thus:

Example (32): To find the tangent of 5° 37′, set the moveable index to the sine (thin type) of 5° 37′, the vernier index to its cosine. Make the *latter* read the end of the second scale when the moveable index reads 9834 on the first scale: the tangent is, therefore 0.09834.

To find the angle from a given tangent, reverse the above operation, and turn the cylinder until both indices read, one, the sine the other the cosine of the *same angle*, which is the angle required.

Functions of Angles greater than 90°: The functions of an angle greater than 90° have the same absolute values as some angle between 0° and 90°. They are connected by the following formulæ, which are the most convenient to remember.

$$\text{Sin. } A = \mp \text{ sin. } (180 \pm A) = \pm \text{ sin. } (360 \pm A)$$
$$\text{Cos. } A = - \text{ cos. } (180 \pm A) = + \text{ cos. } (360 \pm A)$$
$$\text{Tan. } A = \pm \text{ tan. } (180 \pm A) = \mp \text{ tan. } (360 \pm A)$$

The above formulæ are worked out in the following table, which will be found useful.

sin. cos. tan.	+ + +	+ − −	− − +	− + −
	0° 10° 20° 30° 40° 50° 60° 70° 80° 90°	180° 170° 160° 150° 140° 130° 120° 110° 100° 90°	180° 190° 200° 210° 220° 230° 240° 250° 260° 270°	360° 350° 340° 330° 320° 310° 300° 290° 280° 270°
	1st Quad.	2nd Quad.	3rd Quad.	4th Quad.

Thus the sine, cosine or tangent of 123°, for instance, will be the same as the sine, cosine or tangent of 57°, its complement to 180°. In many computations, the algebraical sign of the function must be taken into consideration: they are shown at the top of the table.

Solution of Spherical Triangles: Spherical triangles are easily solved by means of the rule, as the following examples will show:

Example (33): A star whose declination = 8° 31′ 45″ S is observed to be 20° 18′ above the horizon when its hour angle is 20$^{h.}$ 19$^{m.}$ 42$^{s.}$. Find its azimuth.

The formulæ is $\sin. A = \dfrac{\sin. t, \cos. d}{\cos. h}$, where A is the azimuth, d the declination and h the altitude of the star.

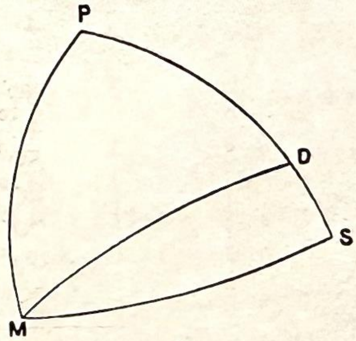
Substituting for the symbols, and reducing the hour angle to degrees, we have $\sin. A = \dfrac{\sin. 304° \; 55' \; 30'', \cos. 8° \; 31' \; 45''}{\cos. 20° \; 18'}$

Now a glance at the table shows that 304° 55′ 30″ is equivalent to 55° 4′ 30″, and that sin. t is negative. The above formulæ reduces itself to $\text{Sin.} A = \dfrac{-\sin. 55° \; 4' \; 30'', \cos. 8° \; 31' \; 45''}{\cos. 20° \; 18'}$

(*a*) Set the vernier index to sin. 55° 4′ 30″ (thin type)
(*b*) Set the moveable index to cos. 20° 18′ (heavy type)
(*c*) Turn cylinder until the moveable index reads 8° 31′ 45″ (heavy type)
(*d*) The vernier index reads the sine of 59° 50′ (thin type).

Hence $A = 59° 50'$; but as its sine is negative, the true azimuth is the complement of that angle to 360 (see table) or $A = 300° 10'$.

Example (34): From the *Nautical Almanack* for 1908:
To find the true distance between Regulus and the Moon at 4 p.m. Greenwich mean time on Jan. 17th, 1908.



The two sides of the spherical triangle are the polar distances of the Moon, $M\ P$, and of the Star, $P\ S$; the contained angle P being the difference of their Right Ascensions. $M\ D$ is a great circle through the moon M perpendicular to $P\ S$.

The formulæ are:

Tan. $P\ D$ = tan. $M\ P$, cos. P - - - (i)
Cos. $M\ S$ = cos. $S\ D$, cos. $P\ M$, sec. $P\ D$ - (ii)

The data from the *Nautical Almanac* gives:

$P\ S = 77° 35'$
$P M = 67° 22'$
$P = 2^{h.} 52^{m.} 3^{s.}$, or in arc, $43° 0' 45''$

Substituting in the first formulæ, we have:

$$\text{Tan.}\ P D = \frac{\text{sin. } 67° 22', \text{ cos. } 43° 0' 45''}{\text{cos. } 67° 22'}$$

(*a*) Set the vernier index to sin. 67° 22′ (thin type)
(*b*) Set the moveable index to cos. 67° 22′ (heavy type)
(*c*) Turn the cylinder until the moveable index reads cos. 43° 0′ 45″ (heavy type)
(*d*) Set the moveable index to the origin
(*e*) Find the angle whose sin. (thin type) is shown by the moveable index while its cosine (heavy type) is shown by the vernier index.

This angle $P\ D$ is 60° 19′.

Now $S D = P S - P D = 17° 16'$: turning now to the second formulæ, we have, substituting their values for the symbols:

$$\text{Cos. } M\ S = \frac{\text{cos. } 17° 16', \text{ cos. } 67° 22'.}{\text{Cos. } 60° 19'.}$$

(*a*) Set the vernier index to cos. 17° 16′ (heavy type)
(*b*) Set the moveable index to cos. 60° 19′ (heavy type)
(*c*) Turn the cylinder until the moveable index reads 67° 22′ (heavy type)
(*d*) The vernier index reads cos. 42° 7′ (heavy type)
Hence the required distance MS = 42° 7′.

Powers and Roots of Trigonometrical Functions: The above examples show that trigonometrical functions are dealt with in exactly the same way as numbers on the arithmetical scales: it is clear, therefore, that the rules which apply to powers and roots of numbers will be equally applicable to powers and roots of trigonometrical functions. Thus:

Example (35): To solve the equation

$$\text{Sin.}^2\ \theta = \text{sin.}\ \phi, \text{ when } \phi = 52^\circ\ 30'.$$

Guess approximately the sine situated at half of the distance on the scale between the origin and the sine of 52° 30′: say it is sin. 60°. It would certainly be guessed much nearer, even by a beginner; but a large error is supposed to have been made in order to illustrate the method more fully.

(*a*) Set vernier index to sin. 60°
(*b*) Set moveable index to origin
(*c*) Turn cylinder so that moveable index reads sin. 60°
(*d*) The vernier index reads 48° 34′: the mean of this and 60° 0′, the original setting is 63° 11′ 30″
(*e*) Repeat the above operation with 63° 11′
When the vernier index is set to 52° 30′, we find the new reading of the moveable index 62° 45′; the mean of which is 62° 58′
Hence we have $\text{sin.}^2\ 62^\circ\ 58' = \text{sin.}\ 52^\circ\ 30'$ or $\theta = 62^\circ\ 58'$.

Logarithmic Functions: Are determined by first finding their numerical values, the required logarithms being obtained by the methods already described.

COMBINATION OF ARITHMETICAL AND ANGULAR SCALES.

The chief advantage of the Co-ordinate Slide Rule lies in the facility with which numbers can be multiplied or divided by the trigonometrical functions. The mode of procedure is exactly similar to that which has been described repeatedly. For example, to multiply a number by the sine of an angle,

(*a*) Set the vernier index to the number
(*b*) Set the moveable index to the origin
(*c*) Turn the cylinder until the moveable index reads the given sine
(*d*) Read the result at the vernier index

The rules for the position of the decimal point are as before.

Example (36): Prove that

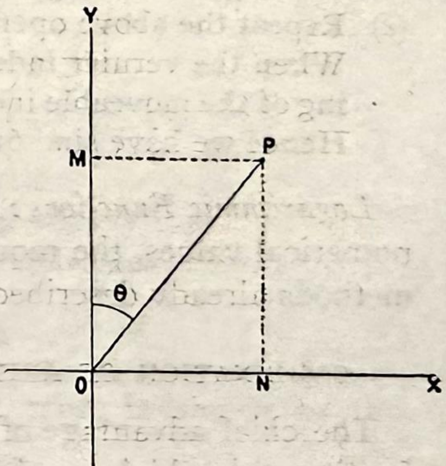
$$2\ \text{sin.}\ A,\ \text{cos.}\ A = \text{sin.}\ 2\ A, \text{ when } A \text{ is } 25^\circ\ 15'.$$

(*a*) Set the vernier index to 200
(*b*) Set the moveable index to the origin
(*c*) Turn the cylinder until the moveable index points to the sine of 25° 15′
(*d*) Set the moveable index to the origin
(*e*) Turn the cylinder until the moveable index points to the cos. of 25° 15′
(*f*) The vernier index points to the sine of 50° 30′, which is twice the given angle, 25° 15′

Example (37): Evaluate the formula $\frac{a \text{ sin. } A \text{ sin. } B}{\text{sin. } (B-A)}$, where
$a = 967$
$B = 16° \ 43'$ $A = 7° \ 5'$ $B - A = 9° \ 38'$
(*a*) Set the vernier index to 967
(*b*) Set the moveable index to sin. 9° 38′, *i.e.*, dividing by sin. $(B-A)$
(*c*) Turn the cylinder until the moveable index points to sin. 7° 5′
(*d*) Set the moveable index to the origin
(*e*) Turn the cylinder until the moveable index reads sin. 16° 43′
(*f*) Read the result 205.2 at the vernier index.

Co-ordinates: If through a point O two lines, $O\,Y$, $O\,X$, are drawn perpendicular to each other, the co-ordinates of a point P relatively to O are $O\,M$, $O\,N$. If the distance $O\,P$, and the angle which $O\,P$ makes with the fixed line $O\,Y$ are given,

$O\,M = O\,P \cos.\theta$; and $O\,N = O\,P \sin.\theta$.



In Land surveying the lines $O\,Y$ and $O\,X$ are taken as North to South and West to East respectively; and the angle θ is called the "bearing" of the line $O\,P$. The co-ordinate $O\,M$ is called the "latitude," and $O\,N$ the "departure," of point P.

To find the co-ordinates of a point when its bearing and distance are given is therefore merely to multiply the given distance into the cosine and sine of the bearing.

In order to avoid mistakes, when many such co-ordinates are computed, the word "Lat." is printed in heavy type followed by the word "Cos."; and the words "Dep." and "Sin." are printed in thin type at the origin.

When the bearing is greater than 90°, the table of equivalent angles is used; and the equivalent angle is called by Surveyors the "reduced bearing" of the line.

Example (38): To find the co-ordinates of a line 768 links long at a bearing of 67° 32′.

(*a*) Set the vernier index to the length of the line, 768
(*b*) Set the moveable index to the origin
(*c*) Turn the cylinder until the moveable index points to cos. 67° 32′ (or latitude, heavy type)
(*d*) Read the vernier index which gives 293.5 links, the "latitude"
(*e*) Turn the cylinder until the moveable index points to sin. 67° 32′ (or Departure, thin type)
(*f*) Read the vernier index which gives 710 links, "the Departure."

Example (39): To find the latitude and departure of a line 487 links long, at a bearing of 176° 19′. The equivalent angle or reduced "bearing" is 3° 41′, as this angle is smaller than 5° 40′ (see page 25),

(*a*) Set the vernier index to 487 on the second arithmetical scale
(*b*) Set the moveable index to the origin
(*c*) Turn the cylinder until the moveable index reads cos. 3° 41′ (lat, heavy type)
(*d*) Read the vernier index, which gives 486 links the latitude required
(*e*) Turn the cylinder until the moveable index reads sin. 3° 41′ (Dep. thin type)
(*f*) Read the vernier index which gives 3128. Now, as this is read on the first scale, in accordance with the rule as to the position of the decimal point, this result must be read as 31.3 links: this is the departure required.

Solution of Plane Triangles:: A little consideration will show that the method of finding co-ordinates given above is equivalent to the solution of a right-angled triangle: the co-ordinates $O\,M$, $O\,N$ being equal to the sides $O\,M$, $M\,P$, of the triangle $O\,M\,P$. An example is, therefore, unnecessary.

To solve oblique-angled triangles reduces itself to the working out of an equation in which the length of sides and functions of the given angles are factors. To take the case which occurs most frequently in practice:

Example (40): Given $c = 79.06$ chains $A = 41° 13'$; $B = 67° 28'$, hence $C = 71° 19'$. Find the two other sides. The formulæ are: $b = \frac{c \text{ sin. } B}{\text{sin. } C}$; $a = \frac{c \text{ sin. } A}{\text{sin. } C}$

The computation is performed as follows:

(*a*) Set the vernier index to 7906
(*b*) Set the moveable index to sin. 71° 19'
(*c*) Turn the cylinder until the moveable index points to sin. 67° 28'
(*d*) Read the vernier index, which gives $77.09 = b$
(*e*) Turn the cylinder until the moveable index points to sin. 41° 13'
(*f*) Read the vernier index, which gives $54.99 = a$

The reader should now be sufficiently practised in the use of the rule to be able to perform any ordinary computation by applying the methods which have been described at length. In some cases, however, a certain amount of time may be saved by the use of special methods, which must be devised to suit the particular problem in hand. To do this it is necessary that he should make himself thoroughly acquainted with the principles upon which the Co-ordinate Slide Rule has been designed. They are set forth concisely in the following chapter.

CHAPTER IV.

LOGARITHMS AND LOGARITHMIC SCALES.

Theory of Indices: Definitions: The "index" of a number indicates the number of times the number is to be multiplied by itself; thus:

$$a^2 = a \times a$$
$$a^3 = a \times a \times a$$
$$a^n = a \times a \times a \times a \text{..........} n \text{ times.}$$

By trial it can be proved that the index n may be any number, integral or fractional. If fractional, it represents the n^{th} root of the number. For instance: $a^{\frac{1}{3}} = \sqrt[3]{a}$; $a^{\frac{1}{2}} = \sqrt[2]{a}$; $a^{\frac{1}{n}} = \sqrt[n]{a}$

The notation $a^{\frac{m}{n}}$ represents $(a^m)^{\frac{1}{n}}$ or $\sqrt[n]{a^m}$

When a^m is raised to the n^{th} power or multiplied by itself n times, it becomes a^{mn}.

If the index n is negative it represents the reciprocal of the number raised to the n^{th} power; thus:

$$a^{-2} = \frac{1}{a^2};\ a^{-3} = \frac{1}{a^3};\ a^{-n} = \frac{1}{a^n}$$

If $n = 0$ the number is always equal to unity, or $a^0 = 1$.

Properties of Indices:

(1) The product of the same number raised to different powers is the sum of the indices of these powers:

$$a^3 \times a^2 = a \times a \times a \times a \times a = a^5 = a^{3+2}.$$

This is expressed symbollically by $a^m \times a^n = a^{m+n}$.

(2) The quotient of a number raised to a given power, by the same number raised to another power, is the difference of their indices.

$$\frac{a^3}{a^2} = a^3 \times a^{-2} = a^{3-2} = a^1 = a, \text{ or, generally, } \frac{a^m}{a^n} = a^{m-n}.$$

(3) From the above definitions, the n^{th} root of a number raised

to any power m is that number raised to a power such that its index m is divided by n.

$$\sqrt[2]{a^4} = a^{\frac{4}{2}} = a^2 ; \quad \sqrt[3]{a^5} = a^{\frac{5}{3}} ; \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

(4) A number already raised to a given power m, is further raised to another power n by multiplying the indices together, $(a^2)^3 = a^2 \times a^2 \times a^2 = a^6 = a^{2\times3}$; generally, $(a^m)^n = a^{mn}$.

The proof of these statements will be found in any treatise on Algebra. The above properties must be well understood before passing on to logarithms.

Logarithms: The logarithm of a number to a given base may be defined as the power to which that base must be raised to be equal to that number. Thus,

if $a^x = m$, x is the logarithm of m to the base a; again,
if $a^y = n$, y is the logarithm of n to the same base.

These relations are expressed as follows: log. $m = x$, log. $n = y$.

The logarithms which are generally used are called "common" logarithms. They are computed to the base $a = 10$. Consider the following identities:

$$\begin{aligned} 10^3 &= 1000 \\ 10^2 &= 100 \\ 10^1 &= 10 \\ 10^0 &= 1 \\ 10^{-1} &= 0.1 \\ 10^{-2} &= 0.01 \quad \text{etc.} \end{aligned}$$

From the definition of a logarithm we see that the logarithm of

$$\begin{aligned} 1000 &= 3 \\ 100 &= 2 \\ 10 &= 1 \\ 1 &= 0 \\ 0.1 &= -1 \\ 0.01 &= -2 \quad \text{etc.} \end{aligned}$$

A little consideration will show that the logarithm of any number between 1 and 10 must lie between 0 and 1, that is, it must be some fraction. Suppose we compute by appropriate methods the value of x in the equation

$10^x = 2$ and we find $x = 0.30103...$ or $10^{0.30103...} = 2$
then the logarithm of $2 = 0.30103...$

A table of logarithms is the result of the solution of a similar equation for each successive number: extracting from it the logarithms of the simple numbers from 1 to 10 we find:

Numbers.		Logarithms.
1	=	0.00000
2	=	0.30103
3	=	0.47712
4	=	0.60206
5	=	0.69897
6	=	0.77815
7	=	0.84510
8	=	0.90309
9	=	0.95424
10	=	1.00000

These logarithms will have their decimal parts the same as for numbers 10, 100, 1000, etc., 10^n larger: for, taking the logarithm of 200 for instance

$$200 = 2 \times 100 = 2 \times 10^2$$

Hence from the theory of indices,

$$200 = 10^{0.30103...} \times 10^2 = 10^{0.30103+2}$$

$$\text{therefore log. } 200 = 0.30103 + 2 = 2.30103$$

The decimal part of a logarithm is always positive and is called the *mantissa*. The integral part of the logarithm is called the "characteristic" because it indicates the order of magnitude of the number.

The properties of logarithms are, and, of course must be, those of indices, since a logarithm is nothing but the index of a number.

Referring back to those properties, we find that:

(*a*) The logarithm of 1 is 0

(*b*) The logarithm of a product is the sum of the logarithms of its factors, thus

$$2 \times 3 = 6$$

$$\text{now } 2 = 10^{0.30103}\text{; hence log. of } 2 = 0.30103$$

$$3 = 10^{0.47712}\text{; hence log. } 3 = 0.47712$$

adding these logarithms together, we have 0.77815

This should be the log. of 6: consulting the table, we find that it is so. The computation may be written as

$$2 \times 3 = 10^{0.13103} \times 10^{0.47712} = 10^{0.30103 + 0.47712} = 10^{0.77815} = 6$$

(*c*) The logarithm of any quotient is equal to the log. of the dividend diminished by the logarithm of the divisor. This requires no explanation, it is merely the inverse of the preceding property.

(*d*) The logarithm of any power, integral or fractional of a number is equal to the product of the logarithm of the number and the index of the power. Thus

$$3^2 = 9$$

$$\text{now } 3 = 10^{0.47712}\text{; and since } (a^m)^n = a^{mn},$$

$$(10^{0.47712})^2 = 10^{0.47712 \times 2} = 10^{0.95424} = 9$$

or, in other words,

$$\log.\ 3 = 0.47712$$

$$\log.\ 3^2 = 0.47712 \times 2 = 0.95424$$

$$= \log.\ 9$$

$$\text{hence } 3^2 = 9$$

The same is, of course, true of roots of any index.

Further information on the theory and practical use of logarithms will be found in treatises on Algebra.

Logarithmic Scales : If, from a scale of equal parts, such as *A B*, the logarithms of the numbers 1 to 10 are set out from *C* to *D*, repeated from *D* to *E*, and cyphered as in Fig. 4,

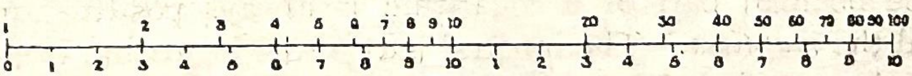


FIG. 4.

it will be clear that the distances corresponding to the numbers on the upper scale will be proportional to their logarithms. If now the legs of a pair of dividers are opened out from 1 to 2 on the upper scale, the distance between them will be equivalent to the log. of 2 on the scale. If this distance is set out from any division, say 4 for instance, the other leg will point to the division marked 8, since the distance corresponding to the log. of 2 on the compasses will have been mechanically added to the distance 1 to 4 corresponding to the log. of 4.

This operation may be written:

$$\log.\ 4 + \log.\ 2 = \log.\ 8 \text{ which corresponds to } 4 \times 2 = 8$$

The same applies to any other distances on the compasses and on the scale.

The second scale *D E* is added for convenience: Thus, if the legs of the compasses are opened to the distance 1 to 5, equal to the log. 5, and one leg is placed on the division 4 on *C D*, the other leg will fall on division 2 on $D\ E$, which would be read as 20, the operation being: log. 5 + log. 4 = log. 20, or $5 \times 4 = 20$

It will readily be seen that, with such a scale, the mechanical addition of logarithms can be performed; and this, of course, corresponds to the multiplication of numbers. Subtraction of logarithms, as easily done, corresponds to the division of numbers.

The arithmetical scales on the Co-ordinate Slide Rule are divided on exactly the same principles. They correspond to a very long logarithmic scale, wound spirally upon a cylinder. The legs of the dividers are represented by the vernier and moveable indices. When one index is set to the origin, and the other to any number, 200 for instance, the distance between them is proportional to the log. of 2. When the cylinder is moved so that the index which was set to the origin points to any number, say 300, for instance, the distance between the indices which has not changed, will add mechanically the log. of 2 to the log. of 3, and will necessarily point to the number 600, and the multiplication of 3 by 3 will have been accomplished.

The reader should have no difficulty in understanding the different methods of computation already described, in the light of these explanations.

Trigonometrical functions are mere ratios and therefore numbers.

The angular scale has been divided proportionally to the logarithms of these ratios, or to the logarithmic sines and cosines. The indices play exactly the same rôle as in the case of the arithmetical scales. The uniformly graduated scale $A\ B$ has its counterpart $C\ D$ in the graduated scale at the foot of the cylinder, at the end of the third arithmetical scale. As previously explained, it is used for finding the logarithm of any particular number on the arithmetical scale.

The present chapter is one which will amply repay careful study. The reader is strongly recommended to plot a logarithmic scale for himself and to experiment upon it with a pair of dividers. He will find that any difficulty which he may have experienced, at first, in the management of the Co-ordinate Slide Rule, will disappear at once, while his mastery of the theory of that instrument will prove of easy acquirement.

TABLES AND FORMULÆ

FOR USE WITH

BARNARD'S CALCULATING RULES.

	Cubic Ins.	Round Rod 1 ft. long, 1" diam.	Square Bar 1 ft. × 1" × 1".	Plate 1 ft. × 1 ft. × 1".
	lbs.	lbs.	lbs.	lbs.
Brass, cast	·298	2·81	3·58	43·0
,, wire	·308	2·91	3·70	44·4
Bronze	·303	2·86	3·64	43·7
Copper, sheet	·318	2·99	3·81	45·75
,, hammered	·322	3·03	3·86	46·3
Iron, cast	·257	2·42	3·08	37·0
,, wrought	·278	2·62	3·33	40·0
Lead	·412	3·88	4·94	59·3
Steel	·283	2·67	3·40	40·8
Zinc	·252	2·38	3·03	36·3

	Cubic Foot.	Tenacity Sq. Ins.	Mod. Elasticity Sq. In.	Mod. Rupture. Sq. In.
	lbs.	lbs.	lbs.	lbs.
Cast iron	444	16,500	17,000,000	—
Wrought iron	480	65,000	29,000,000	—
Steel bars	490	115,000	35,000,000	—
,, plates	—	80,000	—	—
Elm	34	14,000	1,000,000	7,500
Fir, Red Pine	37	13,000	1,600,000	8,000
,, Spruce	37	12,000	1,600,000	11,000
,, Larch	33	9,500	1,100,000	7,500
,, Yellow Pine	29	—	—	7,000
Oak, English	53	15,000	1,500,000	12,000
,, American	54	10,000	2,000,000	10,000
Teak	48	15,000	2,400,000	15,000

Decimals of a Degree or Hour.						Birmingham Wire Gauge.			
Min.	Deg.	Min.	Deg.	Min.	Deg.	No.	Ins.	No.	Ins.
1	·0167	21	·35	41	·6833	1	·300	21	·032
2	·0333	22	·3667	42	·7	2	·284	22	·028
3	·05	23	·3833	43	·7167	3	·259	23	·025
4	·0666	24	·4	44	·7333	4	·238	24	·022
5	·0833	25	·4167	45	·75	5	·220	25	·02
6	·1	26	·4333	46	·7667	6	·203	26	·018
7	·1167	27	·45	47	·7833	7	·180	27	·016
8	·1333	28	·4667	48	·8	8	·165	28	·014
9	·15	29	·4833	49	·8167	9	·148	29	·013
10	·1667	30	·5	50	·8333	10	·134	30	·012
11	·1833	31	·5167	51	·85	11	·120	31	·01
12	·2	32	·5333	52	·8667	12	·109	32	·009
13	·2167	33	·55	53	·8833	13	·095	33	·008
14	·2333	34	·5667	54	·9	14	·083	34	·007
15	·25	35	·5833	55	·9167	15	·072	35	·005
16	·2667	36	·6	56	·9333	16	·065	36	·004
17	·2833	37	·6167	57	·95	17	·058		
18	·3	38	·6333	58	·9667	18	·049		
19	·3167	39	·65	59	·9833	19	·042		
20	·3333	40	·6667			20	·035		

Multipliers for Converting.

Note.—The converse of these is obtained by dividing by the number instead of multiplying.

Common to hyperbolic log.	2·3026
Feet to links	1·5151
Square feet to square links	2·2957
Acres to square yards	4840
Tons to pounds	2240
Lbs. per sq. in. to lbs. per sq. foot	144
Lbs. avoir. to grains	7000
Cubic feet to gallons	6·2355
Rood masonry 2 ft. thick to cub. yds.	24
Rod brickwork 1′ 1½″ ,, ,,	11·333
Metres to feet	3·2809
Inches to millimetres	25·4
Square metres to square feet	10·764
Square inches to square millimetres	645·14
Cubic metres to cubic feet	35·317
Cubic inches to cubic millimetres	16386
Grammes to grains	15·432
Kilogrammes to lbs.	2·2046
Tons to tonneaux	1·0160
Gallons to litres	4·541
Kilogrammetres to foot lbs.	7·233
Kilogram. on square millimetre to lbs. on square inch	1422
Miles to kilometres	1·6093
Hectares to acres	2·4711
£ to francs	25·22
Francs to pence	9·516
Miles per hour to feet per second	1·467
Knots to feet per second	1·688
Cubic feet of water to lbs.	62·425
,, ,, sea ,,	64·05
One atmosphere to lbs. per sq. inch	14·7
,, ,, ,, ,, foot	2116
,, ,, ,, kilogs. per sq. metre	10333
,, ,, ,, millimetre of mercury	760
,, ,, ,, inches ,, ,,	29·922
,, ,, ,, feet of water	33·9

Decimals of a Foot.

in.	ft.	in.	ft.
⅛	·01041	6· ⅛	·51041
¼	·02083	¼	·52083
⅜	·03125	⅜	·53125
½	·04166	½	·54166
⅝	·05208	⅝	·55208
¾	·0625	¾	·5625
⅞	·07292	⅞	·57292
1·	·08333	7·	·58333
⅛	·09374	⅛	·59374
¼	·10416	¼	·60416
⅜	·11458	⅜	·61458
½	·125	½	·625
⅝	·13541	⅝	·63541
¾	·14583	¾	·64583
⅞	·15625	⅞	·65625
2·	·16666	8·	·66666
⅛	·17707	⅛	·67707
¼	·1875	¼	·6875
⅜	·19791	⅜	·69791
½	·20832	½	·70832
⅝	·21874	⅝	·71874
¾	·22916	¾	·72916
⅞	·23958	⅞	·73958
3·	·25	9·	·75
⅛	·26041	⅛	·76041
¼	·27083	¼	·77083
⅜	·28125	⅜	·78125
½	·29166	½	·79166
⅝	·30208	⅝	·80208
¾	·3125	¾	·8125
⅞	·32292	⅞	·82292
4·	·33333	10·	·83333
⅛	·34374	⅛	·84374
¼	·35416	¼	·85416
⅜	·36458	⅜	·86458
½	·375	½	·875
⅝	·38541	⅝	·88541
¾	·39583	¾	·89583
⅞	·40625	⅞	·90625
5·	·41666	11·	·91666
⅛	·42707	⅛	·92707
¼	·4375	¼	·9375
⅜	·44791	⅜	·94791
½	·45833	½	·95833
⅝	·46875	⅝	·96875
¾	·47916	¾	·97916
⅞	·48958	⅞	·98958
6·	·5	12·	1·

$g = 32·2$ feet seconds.
Unit of heat, 772 ft. lbs.
1 H.P. 550 ft. lbs. per second

Decimals of a Cwt.

qr. lbs.	cwt.	qr. lbs.	cwt.	qr. lbs.	cwt.
1	·0089	1 10	·3393	2 19	·6696
2	·0179	11	·3482	20	·6786
3	·0268	12	·3571	21	·6875
4	·0357	13	·3661	22	·6964
5	·0446	14	·375	23	·7054
6	·0536	15	·3839	24	·7143
7	·0625	16	·3929	25	·7232
8	·0714	17	·4018	26	·7322
9	·0803	18	·4107	27	·7411
10	·0893	19	·4196	3 0	·75
11	·0982	20	·4286	1	·7589
12	·1071	21	·4375	2	·7679
13	·1161	22	·4464	3	·7768
14	·125	23	·4554	4	·7857
15	·1339	24	·4643	5	·7946
16	·1429	25	·4732	6	·8036
17	·1518	26	·4822	7	·8125
18	·1607	27	·4911	8	·8214
19	·1696	2 0	·5	9	·8303
20	·1786	1	·5089	10	·8393
21	·1875	2	·5179	11	·8482
22	·1964	3	·5268	12	·8571
23	·2054	4	·5357	13	·8661
24	·2143	5	·5446	14	·875
25	·2232	6	·5536	15	·8839
26	·2322	7	·5625	16	·8929
27	·2411	8	·5714	17	·9018
1 0	·25	9	·5803	18	·9107
1	·2589	10	·5893	19	·9196
2	·2679	11	·5982	20	·9286
3	·2768	12	·6071	21	·9375
4	·2857	13	·6161	22	·9464
5	·2946	14	·625	23	·9554
6	·3036	15	·6339	24	·9643
7	·3125	16	·6429	25	·9732
8	·3214	17	·6518	26	·9822
9	·3303	18	·6607	27	·9911

Decimals of a Lb.

oz.	lbs.	oz.	lbs.	oz.	lbs.
¼	·0156	5	·3125	10½	·6562
½	·0312	5½	·3437	11	·6875
¾	·0468	6	·375	11½	·7187
1	·0625	6½	·4062	12	·75
1½	·0937	7	·4375	12½	·7812
2	·125	7½	·4687	13	·8125
2½	·1562	8	·5	13½	·8437
3	·1875	8½	·5312	14	·875
3½	·2187	9	·5625	14½	·9062
4	·25	9½	·5937	15	·9375
4½	·2812	10	·625	15½	·9687

Decimals of a Pound.

s. d.	£	s. d.	£
½	·002	1 . 2½	·0604
1	·0041	3	·0625
1½	·0062	3½	·0646
2	·0083	4	·0667
2½	·0104	4½	·0688
3	·0125	5	·0708
3½	·0146	5½	·0729
4	·0167	6	·075
4½	·0188	6½	·0771
5	·0208	7	·0791
5½	·0229	7½	·0812
6	·025	8	·0833
6½	·0271	8½	·0854
7	·0291	9	·0875
7½	·0312	9½	·0896
8	·0333	10	·0916
8½	·0354	10½	·0937
9	·0375	11	·0958
9½	·0396	11½	·0979
10	·0416	2 . 0	·1
10½	·0437	4 . 0	·2
11	·0458	6 . 0	·3
11½	·0479	8 . 0	·4
1 0	·05	10 . 0	·5
½	·0521	12 . 0	·6
1	·0541	14 . 0	·7
1½	·0562	16 . 0	·8
2	·0583	18 . 0	·9

Decimals of a Shilling.

d.	s.	d.	s.
½	·0417	6½	·5417
1	·0833	7	·5833
1½	·125	7½	·625
2	·1667	8	·6667
2½	·2083	8½	·7083
3	·25	9	·75
3½	·2917	9½	·7917
4	·3333	10	·8333
4½	·375	10½	·875
5	·4167	11	·9167
5½	·4583	11½	·9583
6	·5		

D. of Year.

D.	Y.
1	·0027
2	·0055
3	·0082
4	·0109
5	·0137
6	·0164
7	·0192
8	·0219
9	·0246
10	·0274
20	·0548
30	·0821
40	·1095
50	·1369
60	·1643
70	·1917
80	·2191
90	·2465
100	·2739
110	·3013
120	·3287
130	·3561
140	·3834
150	·4108
160	·4382
170	·4656
180	·4930
190	·5204
200	·5478
210	·5752
220	·6026
230	·63
240	·6574
250	·6848
260	·7121
270	·7395
280	·7669
290	·7943
300	·8217
310	·8491
320	·8765
330	·9039
340	·9313
350	·9586
360	·9861

D. of an Acre.

r.	p.	Acre.
	1	·00625
	2	·0125
	3	·01875
	4	·025
	5	·03125
	6	·0375
	7	·04375
	8	·05
	9	·05625
	10	·0625
	11	·06875
	12	·075
	13	·08125
	14	·0875
	15	·09375
	16	·1
	17	·10625
	18	·1125
	19	·11875
	20	·125
	21	·13125
	22	·1375
	23	·14375
	24	·15
	25	·15625
	26	·1625
	27	·16875
	28	·175
	29	·18125
	30	·1875
	31	·19375
	32	·2
	33	·20625
	34	·2125
	35	·21875
	36	·225
	37	·23125
	38	·2375
	39	·24375
1	0	·25
2	0	·5
3	0	·75

$\pi = 3{\cdot}1416$. Surface of Sphere πd^2.

Volume of Sphere $\pi d^3 \div 6$.

Arc equal to radius 57·296°.

Cos A – sin (90 – A). Sec A – 1 ÷ cos A.

Tan A – sin A ÷ cos A Cosec A – 1 ÷ sin A.

Cot A – cos A ÷ sin A Versin A – 1 – cos A.

Natural Sines.

Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
0	0000	0029	0058	0087	0116	0145	3	6	9	12	15	17	20	23	26
1	0175	0204	0233	0262	0291	0320	3	6	9	12	15	17	20	23	26
2	0349	0378	0407	0436	0465	0494	3	6	9	12	15	17	20	23	26
3	0523	0552	0581	0610	0640	0669	3	6	9	12	15	17	20	23	26
4	0698	0727	0756	0785	0814	0843	3	6	9	12	15	17	20	23	26
5	0871	0901	0929	0958	0987	1016	3	6	9	12	14	17	20	23	26
6	1045	1074	1103	1132	1161	1190	3	6	9	12	14	17	20	23	26
7	1219	1248	1276	1305	1334	1363	3	6	9	12	14	17	20	23	26
8	1392	1421	1449	1478	1507	1536	3	6	9	12	14	17	20	23	26
9	1564	1593	1622	1650	1679	1708	3	6	9	12	14	17	20	23	26
10	1736	1765	1794	1822	1851	1880	3	6	9	12	14	17	20	23	26
11	1908	1937	1965	1994	2022	2051	3	6	9	11	14	17	20	23	26
12	2079	2108	2136	2164	2193	2221	3	6	9	11	14	17	20	23	26
13	2250	2278	2306	2334	2363	2391	3	6	8	11	14	17	20	23	25
14	2419	2447	2476	2504	2532	2560	3	6	8	11	14	17	20	23	25
15	2588	2616	2644	2672	2700	2728	3	6	8	11	14	17	19	22	25
Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
16	2756	2784	2812	2840	2868	2896	3	6	8	11	14	17	19	22	25
17	2924	2952	2979	3007	3035	3062	3	6	8	11	14	17	19	22	25
18	3090	3118	3145	3173	3201	3228	3	6	8	11	14	17	19	22	25
19	3256	3283	3311	3338	3365	3393	3	5	8	11	14	16	19	22	25
20	3420	3448	3475	3502	3529	3557	3	5	8	11	14	16	19	22	25
21	3584	3611	3638	3665	3692	3719	3	5	8	11	14	16	19	22	24
22	3746	3773	3800	3827	3854	3881	3	5	8	11	14	16	19	22	24
23	3907	3934	3961	3987	4014	4041	3	5	8	11	14	16	19	21	24
24	4067	4094	4120	4147	4173	4200	3	5	8	11	13	16	19	21	24
25	4226	4253	4279	4305	4331	4358	3	5	8	11	13	16	18	21	24
26	4384	4410	4436	4462	4488	4514	3	5	8	10	13	16	18	21	23
27	4540	4566	4592	4617	4643	4669	3	5	8	10	13	15	18	21	23
28	4695	4720	4746	4772	4797	4823	3	5	8	10	13	15	18	20	23
29	4848	4874	4899	4924	4950	4975	3	5	8	10	13	15	18	20	23
30	5000	5025	5050	5075	5100	5125	3	5	8	10	13	15	18	20	23
Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
31	5150	5175	5200	5225	5250	5275	2	5	7	10	12	15	17	20	22
32	5299	5324	5348	5373	5398	5422	2	5	7	10	12	15	17	20	22
33	5446	5471	5495	5519	5544	5568	2	5	7	10	12	15	17	19	22
34	5592	5616	5640	5664	5688	5712	2	5	7	10	12	14	17	19	22
35	5736	5760	5783	5807	5831	5854	2	5	7	9	12	14	17	19	21
36	5878	5901	5925	5948	5972	5995	2	5	7	9	12	14	16	19	21
37	6018	6041	6065	6088	6111	6134	2	5	7	9	12	14	16	18	21
38	6157	6180	6202	6225	6248	6271	2	5	7	9	11	14	16	18	20
39	6293	6316	6338	6361	6383	6406	2	4	7	9	11	13	16	18	20
40	6428	6450	6472	6494	6517	6539	2	4	7	9	11	13	15	18	20
41	6561	6583	6604	6626	6648	6670	2	4	7	9	11	13	15	17	20
42	6691	6713	6734	6756	6777	6799	2	4	6	9	11	13	15	17	19
43	6820	6841	6862	6884	6905	6926	2	4	6	8	11	13	15	17	19
44	6947	6967	6988	7009	7030	7050	2	4	6	8	10	12	15	17	19
45	7071	7092	7112	7133	7153	7173	2	4	6	8	10	12	14	16	18

Natural Sines.

Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
46	7193	7214	7234	7254	7274	7294	2	4	6	8	10	12	14	16	18
47	7314	7333	7353	7373	7392	7412	2	4	6	8	10	12	14	16	18
48	7431	7451	7470	7490	7509	7528	2	4	6	8	10	12	13	15	17
49	7547	7566	7585	7604	7623	7642	2	4	6	8	9	11	13	15	17
50	7660	7679	7698	7716	7735	7753	2	4	6	7	9	11	13	15	17
51	7771	7790	7808	7826	7844	7862	2	4	5	7	9	11	13	14	16
52	7880	7898	7916	7934	7951	7969	2	4	5	7	9	11	12	14	16
53	7986	8004	8021	8039	8056	8073	2	3	5	7	9	10	12	14	16
54	8090	8107	8124	8141	8158	8175	2	3	5	7	8	10	12	14	15
55	8192	8208	8225	8241	8258	8274	2	3	5	7	8	10	12	13	15
56	8290	8307	8323	8339	8355	8371	2	3	5	6	8	9	11	13	14
57	8387	8403	8418	8434	8450	8465	2	3	5	6	8	9	11	12	14
58	8480	8496	8511	8526	8542	8557	2	3	5	6	8	9	11	12	14
59	8572	8587	8601	8616	8631	8646	1	3	4	6	7	9	10	12	13
60	8660	8675	8689	8704	8718	8732	1	3	4	6	7	9	10	11	13

Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
61	8746	8760	8774	8788	8802	8816	1	3	4	6	7	8	10	11	12
62	8829	8843	8857	8870	8884	8897	1	3	4	5	7	8	9	11	12
63	8910	8923	8936	8949	8962	8975	1	3	4	5	6	8	9	11	12
64	8988	9001	9013	9026	9038	9051	1	3	4	5	6	8	9	10	11
65	9063	9075	9088	9100	9112	9124	1	2	4	5	6	7	8	10	11
66	9135	9147	9159	9171	9182	9194	1	2	3	5	6	7	8	9	10
67	9205	9216	9228	9239	9250	9261	1	2	3	4	6	7	8	9	10
68	9272	9283	9293	9304	9315	9325	1	2	3	4	5	6	7	9	10
69	9336	9346	9356	9367	9377	9387	1	2	3	4	5	6	7	8	9
70	9397	9407	9417	9426	9436	9446	1	2	3	4	5	6	7	8	9
71	9455	9465	9474	9483	9492	9502	1	2	3	4	5	5	6	7	8
72	9511	9520	9528	9537	9546	9555	1	2	3	4	4	5	6	7	8
73	9563	9572	9580	9588	9596	9605	1	2	2	3	4	5	6	7	7
74	9613	9621	9628	9636	9644	9652	1	2	2	3	4	5	6	6	7
75	9659	9667	9674	9681	9689	9696	1	1	2	3	4	4	5	6	7

Deg.	0′	10′	20′	30′	40′	50′	1	2	3	4	5	6	7	8	9
76	9703	9710	9717	9724	9730	9737	1	1	2	3	3	4	5	5	6
77	9744	9750	9757	9763	9769	9775	1	1	2	3	3	4	4	5	6
78	9781	9787	9793	9799	9805	9811	1	1	2	2	3	3	4	5	5
79	9816	9822	9827	9833	9838	9843	1	1	2	2	3	3	4	4	5
80	9848	9853	9858	9863	9868	9872	0	1	1	2	2	3	3	4	4
81	9877	9881	9886	9890	9894	9899	0	1	1	2	2	3	3	3	4
82	9903	9907	9911	9914	9918	9922	0	1	1	2	2	2	3	3	3
83	9925	9929	9932	9936	9939	9942	0	1	1	1	2	2	2	3	3
84	9945	9948	9951	9954	9957	9959	0	1	1	1	2	2	2	2	3
85	9962	9964	9967	9969	9971	9974	0	0	1	1	1	1	2	2	2
86	9976	9978	9980	9981	9983	9985	0	0	1	1	1	1	1	1	2
87	9986	9988	9989	9990	9992	9993	0	0	0	0	0	0	0	1	1
88	9994	9995	9996	9997	9997	9998	0	0	0	0	0	0	0	0	0
89	9998	9999	9999	1·000	1·000	1·000	0	0	0	0	0	0	0	0	0

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